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Bullwhip Effect – Logistic Stability Examination in Serial and Arborescent Topologies with Demand Uncertainty and Delay

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Abstract–**The paper analyzes the formation of the bullwhip effect in logistic systems as a significant threat to preserving stability in the face of non-negligible goods transport delay and uncertainty of demand and stock records. The popular order-up-to policy is selected as the method governing the goods flow. A dynamic model of entity interaction is constructed and examined, first, analytically, then in numerical tests for various scenarios of practical significance, e.g., a supply chain with external and local demand signals or real-world European goods distribution system. It has been found that the order-up-to policy does not trigger the bullwhip effect despite the delays in the goods delivery in the nominal operating conditions in supply chains. However, in networked environments, even the basic configuration triggers the bullwhip effect.**

Furthermore, when the stock records are imprecise or erroneous, the bullwhip effect does occur in serial connections and in an amplified form in networked topologies. It leads to economic costs increase and, occasionally, to the loss of system stability. The intensity of the bullwhip effect depends on the type of demand distribution function with the Poisson distribution leading to the most significant order-to-demand variance increase.

Keywords*–*Supply chain control, stability analysis, time-delay systems, transportation networks, bullwhip effect

I. Introduction

Inventory management and goods flow control are decision-centered problems that affect all actors in logistic networks: suppliers, distribution centers, and retailers. The reserves used to meet the unpredictable market demand are replenished with a variety of delivery options. However, the process of goods distribution is influenced by numerous factors not to be neglected in shaping the resource management strategy. For instance, the quantity of merchandise actually obtained from a supplier for sales purposes may differ from the ordered one. Such situation may also occur due to distortions in the process of data collection and information processing, product defects, manufacturing imperfections, or improper transport. Moreover, the stock records registered in the system may be imprecise or erroneous, as caused by human mistakes, spoilage, or theft.

A significant *systemic* distortion in the logistic structures is the bullwhip effect (BE), which manifests itself as an intensified variability of demand translated to the goods ordering signal. This distortion propagates in the supply chain, augmenting the costs of subsequent suppliers. In addition to reduced economic benefits, the BE leads to unnecessary shipments and excessive transport and resource accumulation at the intermediate nodes. Pioneering research concerning the BE was conducted by Forrester [1], and a detailed study of its formation in production-distribution systems was given in [2] and [3]. The main factors impacting the goods flow in distributed architectures were analyzed by Lee et al. in [4] and [5], whereas the BE triggers in the current business context were discussed in [6]–[8]**.** Five leading causes of the BE: run-through demand forecasting, production inadequacy, transit time, batch arrangement, and charge deviations; have been recognized. Besides examining its origins, various researchers sought methods of reducing the BE negative influence on the logistic system performance [9]–[12]. However, they focused on the operations research methods and long-term statistical analysis. A viable alternative is sought in the application of a dynamic system approach and robust control methods [13]–[17]. The BE formation in the systems governed by the order-up-to (OUT) policy has been examined in [18] in the context of serial and arborescent topologies. Practical treatment of systems with arbitrary networked topology is given in [19], while more elaborate cases are discussed in [20]. The preliminary studies concerning the networked OUT policy application for the BE reduction have been reported in [21].

In this paper, the performance of the supply chain, organized in a serial structure with multiple demand points, is analyzed from the dynamic system perspective and described in the state-space model. The goods flow proceeds according to the ordering decisions established via the popular OUT inventory policy, applied autonomously at each node in the chain. The good reflow is subject to a non-negligible time delay. The system cohesion concerning the formation of the BE is examined by tracking the evolution of the stock level, satisfied demands, and order quantity requested by the nodes. As a quantitative measure of *logistic* stability, the order-to-demand variance ratio is selected. Numerous simulation tests support the analytical study. In the experiments, a few probability distributions, typically considered in inventory management systems, have been examined to gauge the system proclivity to the BE formation. Also, the system sensitivity with respect to the measurement error (inaccurate stock level records) has been explored.

In extension to the study conducted in [22], this work expands the numerical studies to the networked topologies examination. Therefore, the networked indicators are introduced to perform the appropriate BE quantification in the

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networked context. The contribution of this work is evaluating the real-world distribution networks having both serial and arborescent topologies. Hence, the real-life European goods distribution network is taken into consideration within numerical studies. The noteworthy results are related to the analysis of improper records with reduced stock level. In contrast to the serial connection case, in the networked environment it leads to significant BE amplification and system is unstable in logistic context.

The paper is organized in the following way. The ordering policy used in the proposed framework is described in Section II. The system model and detailed analytical treatment are given in Section III. Section IV introduces the new BE measures, pertinent in the multi-dimensional context. Section V is devoted to numerical studies. Finally, conclusions and managerial insights are presented in Section VI.

II. Order-Up-To Inventory Policy

In order to refill the stock drained to meet the external (market) demand and inner goods requests enforced at a node in the chain, the OUT policy can be used. The OUT policy indicates to collect the stock at the node warehouse up to a reference level, if the sum of already gathered goods and goods in transit decreases below that level. When the prognosis of future demand evolution is not applied, the order quantity is calculated by the OUT policy according to (see [23] for a comprehensive description of the underlying ordering tactics):

$$u_i(t) = x_i^{ref} - x_i(t) - \theta_i(t), \tag{1}$$

where: x_i^{ref} – the reference level, $i \in \Omega_Z = \{1, \ldots, N\}$ – node index, N – number of nodes, t – an instant of stock review and taking the ordering decision, $x_i(t)$ – on-hand stock level, and $\theta_i(t)$ – the open-order volume, i.e., the goods in transit that have not yet reached node i due to lead-time delay.

The tuning strategies with respect to the policy parameter (reference stock level) selection in the basic constructs are discussed in [24], whereas more advanced topologies are treated in [25].

III. Bullwhip Effect in Supply Chain

A. BE at One Echelon

In the subject literature [26], one can find various measures used to quantify the BE, both in the time and the frequency domain. One of the most popular BE indicators is the order-to-demand variance ratio, as introduced for the two-node interaction by Chen et al. in [27]. This bullwhip indicator (BI) is calculated as:

$$b_1 = \frac{\mathrm{var}[u_1]}{\mathrm{var}[d_1]}, \tag{2}$$

i.e., as the ratio of the variance of generated stock replenishment signal – u_1 – to the variance of demand – d_1 – imposed on that node. Fig. 1 illustrates the variables at one echelon.

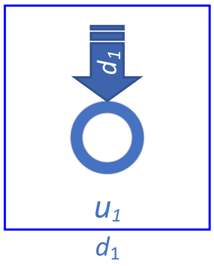

Fig. 1. BE measurement at one echelon: d_1 – demand, u_1 – generated ordering signal, b_1 – BE indicator.

B. Serial Connection – BE with respect to External Demand Only

In this section, a serial connection of N nodes, with an external supplier (having unlimited resources), will be analyzed. The considered situation is depicted in Fig.2, with S_1 – the goods supplier.

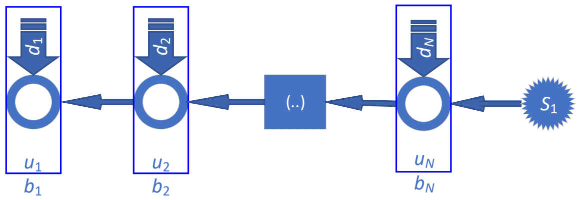

Fig. 2. Serial connection of n nodes: $u_1, u_2, \ldots, u_N$ – ordering signals and $d_1, d_2, \ldots, d_N$ – imposed demands. Resources are provided by external source S_1. The arrows indicate the flow of information.

Equation (2) quantifies the BE formation at a single entity. If $b_1 > 1$, the BE is observed. For a chain structure, considering BE formation with respect to the external demand only, one can formulate the following expression

$$\begin{bmatrix} \mathrm{var}[u_1] \\ \mathrm{var}[u_2] \\ \vdots \\ \mathrm{var}[u_N] \end{bmatrix} = \underbrace{\begin{bmatrix} b_1 & & & \\ & b_2 & & \\ & & \ddots & \\ & & & b_N \end{bmatrix}}_{\mathbf{B}} \begin{bmatrix} \mathrm{var}[d_1] \\ \mathrm{var}[d_2] \\ \vdots \\ \mathrm{var}[d_N] \end{bmatrix}. \tag{3}$$

with the BI in a matrix form:

$$\mathbf{B} = \mathbf{b}^T \mathbf{I}_N, \tag{4}$$

where $\mathbf{I}_N$ is an $N \times N$ identity matrix and $\mathbf{b} = [b_1\ b_2\ \ldots\ b_N]^T$ is a vector grouping the individual BI determined separately at each node.

In the nominal operating conditions, the OUT policy guarantees that (locally) BI is not triggered. We have thus [27, 15]:

$$b_1 = b_2 = \ldots = b_N = 1. \tag{5}$$

Assuming that $d_1, d_2, \ldots, d_N$ are not correlated (given the information about demand imposed at a node, one should generally not infer about the magnitude of demand at other nodes), one can ensure that:

$$\mathrm{var}[u_1 + d_2] = \mathrm{var}[u_1] + \mathrm{var}[d_2] \tag{6}$$

Thus, using (5) and (6), var[u_N] can be calculated as:

$$\begin{aligned}\mathrm{var}[u_1] &= b_1\,\mathrm{var}[d_1] = \mathrm{var}[d_1]\\ \mathrm{var}[u_2] &= \mathrm{var}[u_1 + d_2]\\ &= \mathrm{var}[u_1] + \mathrm{var}[d_2]\\ &= \mathrm{var}[d_1] + \mathrm{var}[d_2]\\ &\vdots\\ \mathrm{var}[u_N] &= \sum_{i=1}^{N} \mathrm{var}[d_i].\end{aligned} \tag{7}$$

C. Serial Connection – BE with respect to External and Internal Demand

In order to conclude about the BE formation, in the circumstances better reflecting a real-world scenario, both internal and external demand signals should be taken into account. The corresponding supply chain structure is depicted in Fig. 3.

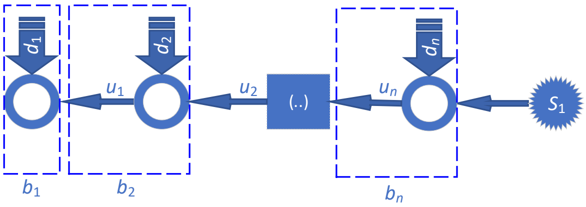

Fig. 3. BE calculation in a serial connection of N nodes with internally (u_1, u_2, …, u_N) and externally (d_1, d_2, …, d_N) imposed demand.

In that case, the BI at the nodes is calculated consecutively as:

$$\begin{aligned}b_1 &= \frac{\mathrm{var}[u_1]}{\mathrm{var}[d_1]}\\ b_2 &= \frac{\mathrm{var}[u_2]}{\mathrm{var}[d_2 + u_1]} = \frac{\mathrm{var}[u_2]}{\mathrm{var}[d_2] + \mathrm{var}[u_1]}\end{aligned} \tag{8}$$

and the variance as

$$\begin{aligned}\mathrm{var}[u_2] &= b_2(\mathrm{var}[d_2] + \mathrm{var}[u_1])\\ &= b_2\,\mathrm{var}[d_2] + b_2\,\mathrm{var}[u_1]\\ &= b_2\,\mathrm{var}[d_2] + b_2 b_1\,\mathrm{var}[d_1]\\ \mathrm{var}[u_3] &= b_3\,\mathrm{var}[d_3] + b_3 b_2\,\mathrm{var}[d_2] + b_3 b_2 b_1\,\mathrm{var}[d_1]\\ &\vdots\end{aligned} \tag{9}$$

Finally, at the last node,

$$\begin{aligned}\mathrm{var}[u_N] &= b_N\,\mathrm{var}[d_N] + b_N b_{N-1}\,\mathrm{var}[d_{N-1}]\\ &\quad + \ldots + b_N \ldots b_1\,\mathrm{var}[d_1]\\ &= \sum_{i=1}^{N}\left(\prod_{j=i}^{N} b_j\right)\mathrm{var}[d_i].\end{aligned} \tag{10}$$

In a vector form:

$$\mathrm{var}[\mathbf{u}] = \begin{bmatrix} b_1 & & & & \\ b_2 b_1 & b_2 & & & \\ b_3 b_2 b_1 & b_3 b_2 & b_3 & & \\ & & & \ddots & \\ \prod_{i=1}^{n} b_i & \prod_{i=2}^{n} b_i & & \prod_{i=n-1}^{n} b_i & b_n \end{bmatrix} \mathrm{var}[\mathbf{d}]. \tag{11}$$

If the OUT policy is applied to manage the flow of goods in the chain, using (5), the ordering signal variance at the last node preceding the external source

$$\mathrm{var}[u_N] = \sum_{i=1}^{N} \mathrm{var}[d_i]. \tag{12}$$

Thus, to allow one to properly quantity the BE formation at a single – ith – echelon, including both internal and external demand, the measure is used:

$$b_i = \frac{\mathrm{var}[u_i]}{\mathrm{var}[d_i] + \mathrm{var}[u_{i-1}]}, \tag{13}$$

where $i \in \Omega_Z$.

Therefore, with the OUT policy in place, the order variance at a node equals the sum of demand variance at all the preceding echelons in the nominal operating conditions. Although the BE is not generated, the system is at the margin of stability. An improper assignment of the reference values of the underlying ordering tactics used in the OUT policy would result in a loss of system stability. Also, an erroneous decision made at one echelon, e.g., caused by imprecise information recorded in the system database, may be the source of instability.

Nevertheless, to evaluate the system proclivity to the BE formation in contemporary systems, one should also consider more complex topologies than a serial chain. An example networked structure is illustrated in Fig. 2, where n_{1-3} represent controlled nodes, $S_{1,2}$ denote external sources, and d_{1-3} is the exogenous demand imposed at the system. $\varepsilon_{1,2}$ – indicate internal demand placed at node n_1 and n_2 from n_3, correspondingly.

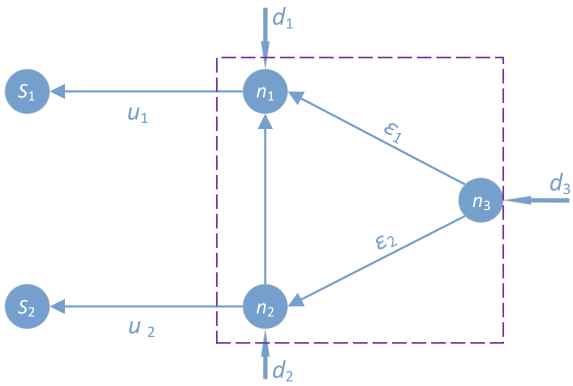

Fig. 4. A five node network.

IV. Bullwhip Effect Quantification in a Networked Environment

A relatively large group of works investigated the BE formation under the OUT policy serial connections. However, the missing aspect is an analysis of how the BE propagates in multi-echelon supply chains when the demand exhibits stochastic variations.

In a serial chain, one may designate the extreme nodes (right and left, accordingly) in the system and use them to determine the BI. There is a limited possibility to follow this approach in a networked environment. The application of indicators for a serial connection is insufficient to quantify the BE in a networked system, even in the basic configuration**,** depicted in Fig. 4. Moreover, equation (5) does not hold in the networked setting. Different quantification methods are therefore needed. The lack of feasible measurement methods, giving the BE quantification in networked topologies, motivated the search for alternatives [32].

In [34] researchers proposed a few networked indicators for the BE measurement, with its upper and lower boundaries defined by demand variance function. For the BE quantification in networked topologies, a vector-based measure will be introduced. As an alternative to focusing on a particular node, demand and external replenishment signals will be considered together. Typically, the demand may be imposed on any node, and any node can generate a replenishment signal for its neighboring nodes or external suppliers.

Taking the Chebyshev distance as the vector measure, one may quantify the BE in the networked system through

$$\gamma = \max_{i \in \Omega_Z} \left\{ \frac{\text{var}[u_i]}{\text{var}[d_i]} \right\}, \tag{14}$$

where $\forall_{i \in \Omega_Z} d_i > 0$.

Indicator – γ –, informs about presence of the BE in the system, $\gamma > 1$. The BE formation is treated externally, without delving into the decisions made at the nodes. The network is treated as a holistic entity.

The second introduced indicator – γ^{max} – corresponds to the L$_1$ norm (Manhattan-type norm). It is obtained as:

$$\gamma^{\max} = \frac{\sum_{i \in \Omega_u} \text{var}[u_i]}{\sum_{j \in \Omega_d} \text{var}[d_j]}, \tag{15}$$

where $\forall d_i > 0, i \in \Omega_Z$.

$\mathbf{\Omega}_u$ denotes the set of node indices that generate replenishment signals for the external suppliers, and $\mathbf{\Omega}_d$ indicates the set of node indices at which the demand is imposed.

γ^{max} does not directly show the BE intensity. It designates its upper limit for the analyzed network.

V. Numerical Studies

To examine the system performance, the dynamic model of entity interaction was implemented, with the state-space characterization included. In the experiments reported in Section A, two cases were analyzed for a supply chain organized in a serial structure, depicted in Fig. 5. In the first series of simulations, the demand signal is imposed only at the first node in the chain (node 1) – it reflects the situation illustrated in Fig. 2. In the second series of simulations described in that section, all controlled nodes are subjected to the external demand, following the structure from Fig. 3.

Section B is devoted to more elaborate – networked – structures. A dedicated, multithreaded application has been prepared to conduct a comprehensive study of the goods distribution process in those systems. First, the basic networked structure from Fig. 4 is given numerical evaluation. In the second series of tests, a real-world transportation network, shown in Fig. 14, is considered. A related network graph is drawn in Fig. 15. Additionally, instead of a single external supplier existing in a serial connection, the networked environment involves two external supply points.

The main parameters used in the simulation scenarios are grouped in Table I. The variety of system quantities recorded within the nominal case are represented in Table II. The variance of system variables during the stock reduced state are arranged in Table III and, accordingly, the stock increased state system parameters are assorted in Table IV. Accordingly, Tables V-VII are grouping the primary Network A measured system parameters, where Tables VIII-X represent the real-world Network B goods distribution quantified values.

A. Serial Connection

Existing supply chains and goods distribution systems are expected to provide a high customer satisfaction level – a high degree of service. The fill rate, i.e., the fraction of external demand satisfied from the resources available at the nodes, is influenced by selecting the reference stock level. In the tests, the reference x_i^{ref} was set through numerical calculations following the guidelines from [28] so that a 100% fill rate is achieved with minimum holding costs. With the estimate of maximum demand $\mathbf{d}^{max}$ = [10 15 20 17 9 13]^T units, the reference level was set as $\mathbf{x}^{ref}$ = [80 60 235 258 436 262]^T units. For clarity, in Fig. 5, the external supplier is denoted by a star symbol. The dotted arrow symbolizes the initial goods inflow from the external source (node 7), which is then dispatched sequentially among nodes 1–6 to fulfill the demand.

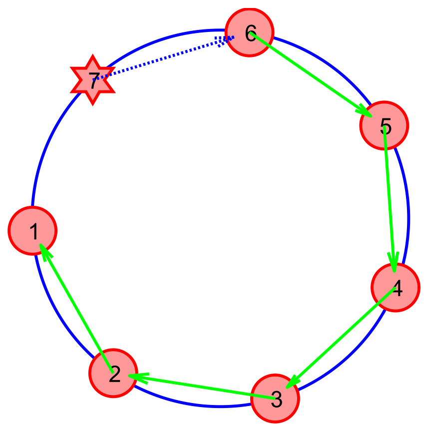

Fig. 5. Serial configuration considered in the numerical study.

The time of a single simulation run was set as 10^4 periods to get a representative data sample. To conclude about the system stability in the circumstances reflecting various real-world scenarios, the demand signal imposed on nodes 1–6 exhibit stochastic variations. In the experiments, the demand is generated by multiplying the estimate of maximum value by a coefficient, as stipulated in Table I. In period *t*, for node *i*, the demand value δ_i is established via a two-step calculation:

$$\psi_i \leftarrow 0.6 * d_i^{\max}, \\ \delta_i \leftarrow round(\phi(\psi_i)), \tag{16}$$

round means rounding to the nearest integer, ϕ is the distribution function, and ψ_i is the input parameter.

To investigate the fundamental system response, the on-off signal in examined during the first 100 periods, illustrated in Fig. 5. The satisfied demand, denoted by $h_i(t)$, is given in Fig. 6. In the nominal operating conditions, $h_i(t) = d_i(t)$, implying the fill rate of one. The ordering signal generated to replenish the stock is shown in Fig. 7, and the stock level in Fig. 8. In the time interval from 60 up to 100 *t*, the stock reaches the steady-state corresponding to the maximum demand. This is typical behavior of a correctly implemented OUT policy with a constant demand signal, in the analyzed case $d_i(t) = d_i^{\max}$.

1) Nominal Operating Conditions

The logistic stability was evaluated quantitatively by examining BI (2). The measured variance of d_i, u_i, and x_i at nodes 1–6 in response to demand signal specified in Table I, is given in Table II. The system response to demand with the same variance (and different distribution types) is denoted by sets **A**, **B**, and **C**. The various distribution applied to generate the external demand signal: *Poisson*, *Gamma*, *Uniform*, and *Normal*, marked by the same color and letter in Table I, resulted in the equivalent system variables. The lead-time delays from node *i* to *j*, where $i,j \in \Omega_Z$ are grouped in the vector $\boldsymbol{\Lambda} = [6_{1\text{-}2}\ 1_{2\text{-}3}\ 4_{3\text{-}4}\ 3_{4\text{-}5}\ 5_{5\text{-}6}\ 2_{6\text{-}7}]^T$. The situation of supplying the external source is omitted by restricting the dimension to $(N + S, N)$, where *S* is the number of external sources. The numerical studies confirm that for the examined supply chain, relations (5) and (7) do hold. In the nominal operating conditions, i.e., when the stock records are correctly registered, one can ensure precise OUT policy realization and relation (5) is satisfied. The system remains stable, regardless of demand uncertainty, type of applied distribution function, and delivery delays.

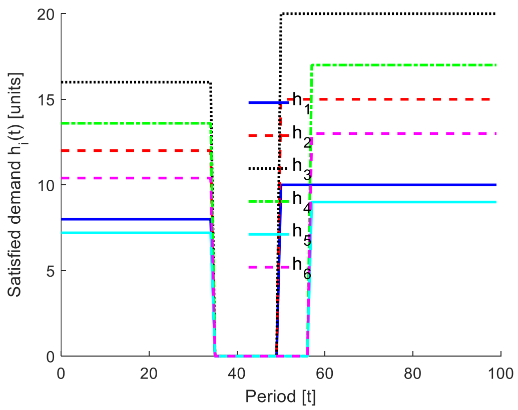

Fig. 6. Satisfied demand imposed on nodes 1–6 (equal to the imposed one).

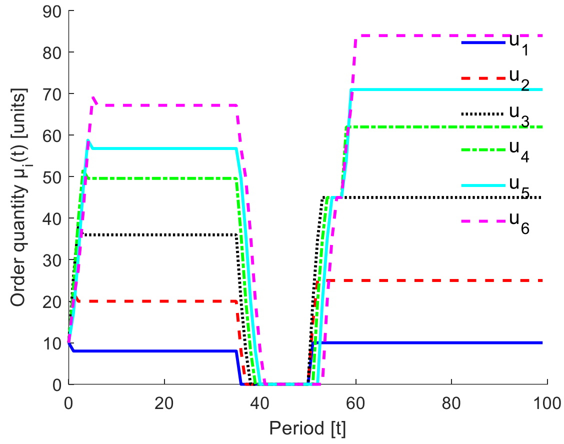

Fig. 7. Order quantity generated at nodes 1–6, nominal case.

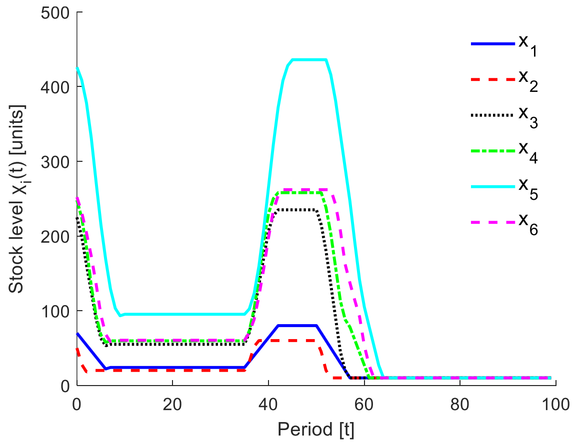

Fig. 8. Stock level at nodes 1–6, nominal case.

TABLE I. PARAMETERS OF SIMULATION SCENARIOS

No	*Probability Distribution*	*Input parameter*	*Set*
1	Poisson	**mean** = 0.6 * d_i^{max}	**A**
2	Poisson	**mean** = 0.1 * d_i^{max}	**B**
3	Poisson	**mean** = 0.3 * d_i^{max}	**C**
4	Gamma	**α** = 0.6 * d_i^{max}	**A**
5	Gamma	**α** = 0.1 * d_i^{max}	**B**
6	Gamma	**α** = 0.3 * d_i^{max}	**C**
7	Uniform	**max** = 0.6 * d_i^{max}	**A**
8	Uniform	**max** = 0.3 * d_i^{max}	**B**
9	Uniform	**max** = 0.5 * d_i^{max}	**C**
10	Normal	**mean** = 0.1 * d_i^{max}, **std** = 0.6 * d_i^{max}	**A**
11	Normal	**mean** = 0.3 * d_i^{max}, **std** = 0.3 * d_i^{max}	**B**
12	Normal	**mean** = 0.6 * d_i^{max}, **std** = 0.6 * d_i^{max}	**C**

TABLE II. VARIANCE OF SYSTEM VARIABLES (NOMINAL CASE)

Parameter	Measured value						Set
node i	1	2	3	4	5	6	
var[d_i]	3.2	6.7	12.1	8.6	2.6	5.2	A
var[u_i]	3.2	9.9	22.1	30.3	33	37.9	A
var[x_i]	22.7	19.8	107.8	119.3	199.4	113.3	A
var[d_i]	0.9	1.7	3.1	2.3	0.6	1.4	B
var[u_i]	0.9	2.7	5.8	8.1	8.8	10.2	B
var[x_i]	6.2	5.3	29.8	33.5	61.1	32.1	B
var[d_i]	2.5	4.7	6.9	5.3	2.3	3.6	C
var[u_i]	2.5	7.2	14.2	19.5	22	25.7	C
var[x_i]	14	12.3	75.4	86.4	167.4	82	C

2) Imprecise Stock Level Records

Occasionally, the stock records registered in the system do not match the actual level of accumulated resources due to human errors, spoilage, or theft. Moreover, owing to distortions in information processing, product imperfections, manufacturing defects, or improper transport, the usable quantity of goods obtained from suppliers for sales purposes could vary from the requested one [29]. In the evaluated system, the information uncertainty was simulated by introducing a random error in the estimates concerning the stock level $x_i(t)$ given in (1).

Two cases were examined – when the stock level employed in establishing the order quantity by the OUT policy (1) is randomly decreased, and a complementary case of (random) stock increase. The actual stock level records fluctuate in the range [a b] according to

$$\begin{aligned} v &\leftarrow (b-a)*\sigma + a\,, \\ x(t) &\leftarrow x(t)*v, \end{aligned} \tag{17}$$

where σ indicates the value obtained from a uniform distribution and v denotes an error intensity coefficient.

In the first scenario, the stock level was reduced to 75–80% of the initial value. Consequently, as presented in Fig. 9, the order quantity begins to fluctuate, yet all the demands are fulfilled. Surprisingly, as recorded in Table III, the BI has dropped below 1 at every second node in cases **A** and **C**, whereas in set **B** the BI has decreased below 1 only at the second and sixth node. It means that a lower variance of the input demand signal reduces the BE in the system.

The demand variations have been throttled down the supply chain, yet the dampening intensity correlates with the type of distribution. The most significant variance reduction was observed for the Poisson distribution. The stock level (Fig. 10) reaches a level exceeding the expected minimum, bringing higher holding costs with respect to the nominal case.

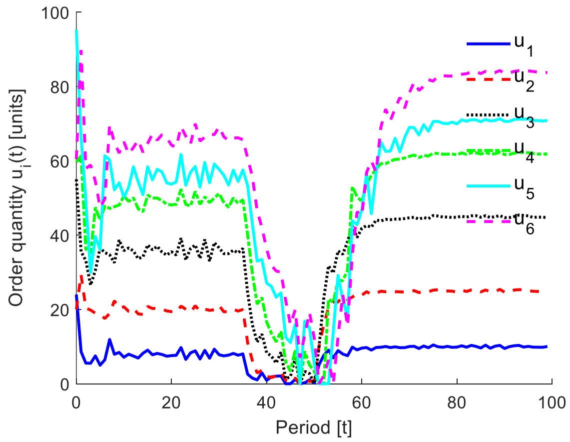

Fig. 9. Order quantity generated at nodes 1–6, under improper records with reduced stock level.

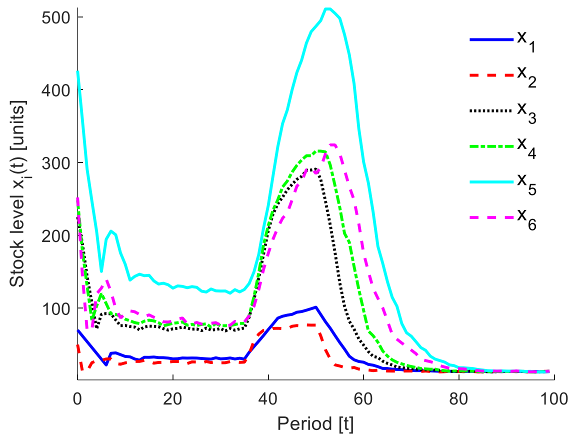

Fig. 10. Stock level at nodes 1–6, under improper records with reduced stock level.

An interesting finding is related to an opposite case study when the stock level value used in the computations was increased to 120–125%, with respect to its actual level in inventory. Contrary to the earlier findings, the BI increases significantly at all nodes in sets **A**, **B**, and **C**, as specified in Table IV. The order quantity varies considerably (Fig. 11), together with the stock level (Fig. 12), which leads to unsatisfied demand (Fig. 13). Since $b_i > 1$, the system does not maintain stability in the logistic sense. To alleviate the gravity of the BE, one may consider alternative control strategies, e.g., proportional OUT policy [26], or a robust LQ-based one [30].

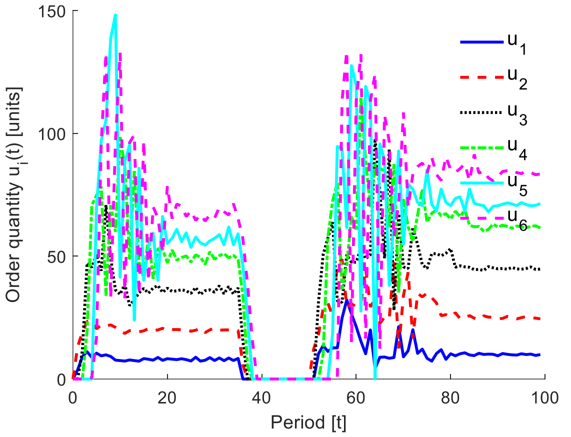

Fig. 11. Order quantity generated at nodes 1–6, under improper records with increased stock level.

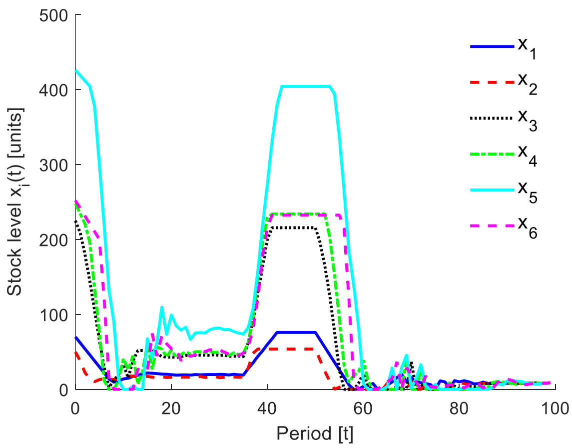

Fig. 12. Goods stored at nodes 1–6, under improper records with increased stock level.

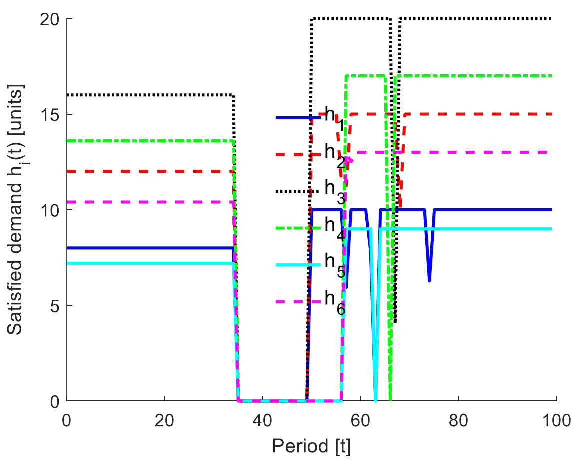

Fig. 13. Satisfied demand at nodes 1–6, under improper records with increased stock level.

TABLE III. VARIANCE OF SYSTEM VARIABLES (STOCK REDUCED)

Parameter	**Measured value**						**Set**
node i	**1**	**2**	**3**	**4**	**5**	**6**	
var[d_i]	3.2	6.8	12.2	8.8	2.5	5.2	**A**
var[u_i]	3.9	6.6	23.9	25.3	45.3	25.6	**A**
var[x_i]	23	20.8	109.6	111.3	172.1	108.5	**A**
b_i	1.23	0.61	1.23	0.77	1.62	0.51	**A**
var[d_i]	0.9	1.7	3.1	2.3	0.6	1.4	**B**
var[u_i]	2.4	2.4	13.9	14.4	33.9	16.8	**B**
var[x_i]	7.8	7	35.5	44.4	76.2	62	**B**
b_i	2.57	0.58	2.52	0.88	2.26	0.48	**B**
var[d_i]	1.8	4.1	7.2	5.3	1.5	3.1	**C**
var[u_i]	1.5	3.6	7.9	8.8	8.8	7.7	**C**
var[x_i]	14.8	11.5	71	65	118.6	44.6	**C**
b_i	0.81	0.64	0.73	0.67	0.85	0.65	**C**

TABLE IV. VARIANCE OF SYSTEM VARIABLES (STOCK INCREASED)

Parameter	**Measured value**						**Set**
node i	**1**	**2**	**3**	**4**	**5**	**6**	
var[d_i]	3.2	6.7	12.3	8.7	2.6	5.2	**A**
var[u_i]	5.5	19.1	54	99.3	163.8	253.7	**A**
var[x_i]	24.2	25.2	149.7	259.8	603	604.4	**A**
b_i	1.74	1.56	1.72	1.59	1.61	1.50	**A**
var[d_i]	0.9	1.7	3.2	2.3	0.6	1.4	**B**
var[u_i]	2.2	6	17.9	31.5	55.4	81.6	**B**
var[x_i]	7.4	7.9	46	83	212.1	194.5	**B**
b_i	2.32	1.54	1.94	1.56	1.72	1.43	**B**
var[d_i]	1.9	4.1	7.2	5.2	1.5	3.1	**C**
var[u_i]	3.1	11.5	30	56	94.2	142.2	**C**
var[x_i]	14.7	15.1	91.7	152.1	373.7	319.9	**C**
b_i	1.65	1.6	1.61	1.59	1.64	1.46	**C**

B. Networked Configuration

This section provides a detailed examination of networked system logistic stability. The indicators introduced in Section IV are used to quantify the BE. Therefore, in addition to the classical BE investigation methods (applicable in the serial connection studied in Section A), here, those new measures are applied. Initially, Network A is studied to examine the prepared model efficiency in the basic networked configuration. In the second series of tests, a real-world distribution Network B is analyzed from the perspective of the BE manifestation in the complex topology.

The considered networks are characterized by various parameters related to the topology complexity level. In [31], a few indicators for judging the complexity of networked configurations have been proposed. Two measures are selected for the considered networks: the number of links per controlled node (δ) and the total number of nodes ($N + S$).

The topologies are described by:

- The channel utilization rate β, which states how much of the current lot requested by node n_j is to be obtained from node n_i, $\beta_{ij} \in [0, 1]$,
- The lead-time delay Λ is the delay in order realization and shipment from node n_i to node n_j, $\Lambda_{ij} \in [1, L]$, where L indicates the maximum delay between any two directly connected nodes.

Each transportation link is utilized according to even lot distribution over the number of nodes. Hence, channels of both Networks A and B are allocated equally, following the standard practice in the literature [33]. The objective is to determine the system stability and validate the BE existence.

1) Basic Network Configuration – Nominal Operating Conditions

Fig. 5 depicts the topology of Network A, an example of a basic configuration. Network A with a centralized topology comprises two external sources (orange circles) and three controlled nodes (blue circles). The numbers displayed over the arrows denote the channel utilization rate θ_{ij} and the lead times ζ_{ij} associated with the routes.

The logistic stability was evaluated quantitatively by examining BI (2) for each controlled node, the BE value in the networked context (14) and the BE maximum boundary for the entire network treated as a holistic, multi-input multi-output entity (15). The measured variance of d_i, u_i, and x_i at nodes 1–3 in response to demand signal given in Table I, is provided in Table V. The lead-time delays from node i to j, where $i,j \in \Omega_Z$ are grouped in the vector $\mathbf{\Lambda} = [2_{4\text{-}1}\ 4_{1\text{-}2}\ 4_{1\text{-}3}\ 4_{5\text{-}2}\ 4_{2\text{-}3}]^T$. The situation of supplying the external source is excluded by limiting dimension to (N + S, N), where S indicates the number of external sources.

The conducted experiments confirm that even for the basic network representation relations (5) and (7) do not hold anymore in Network A depicted in Fig. 14. In the nominal operating conditions, i.e., when the stock records are correctly registered, with the classical BE measures, one cannot determine the system stability properly. Hence, the networked indicators introduced in Section IV are used. Surprisingly, regardless of the type of stochastic distribution used to generate the demand signal of distribution function input parameters, inside evaluated sets **A**, **B**, and **C,** the networked BI $\gamma = 1$. One may notice that this is an expected behavior of accurately implemented OUT policy. Hence, it may indicate that the proposed networked BE indicators enable one to properly measure the BE in the system. Hence, the examined configuration, having the number of connections per controlled node ($\delta = 1.67$) and the total number of nodes ($N + S = 5$), allowed for significant BE mitigation. Therefore, considering the new networked indicators, one can assume that the system is almost stable, regardless of demand uncertainty and delivery delays. However, the maximum BI for the following sets **A**, **B**, and **C**, $\gamma^{\max} = [2.52, 2.57, 2.59]$ indicates that the considered network can be further optimized, e.g., through better channel utilization assignment. The customer satisfaction rate is also below full realization for controlled node n_3, which could be improved, e.g., using networked OUT policy [25].

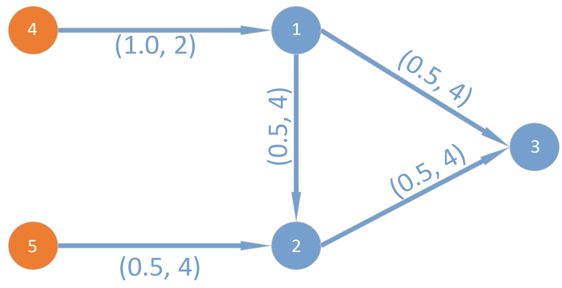

Fig. 14. Graph representation of Network A. The arrows indicate the goods flow direction. The numbers denote β_{ij} and Λ_{ij}.

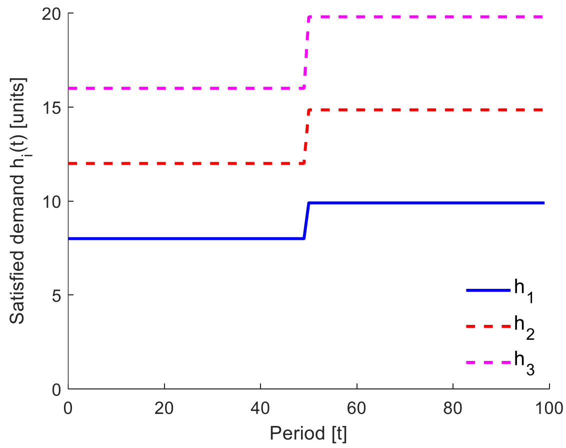

Fig. 15. Satisfied demand at nodes 1–3, nominal case, Network A (equal to the imposed one).

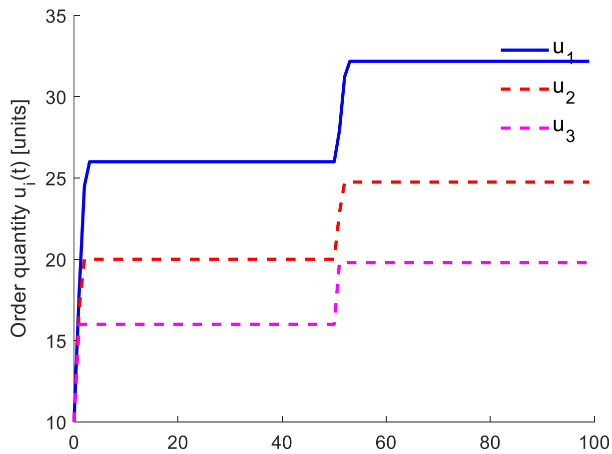

Fig. 16. Order quantity generated at nodes 1–3, nominal case, Network A.

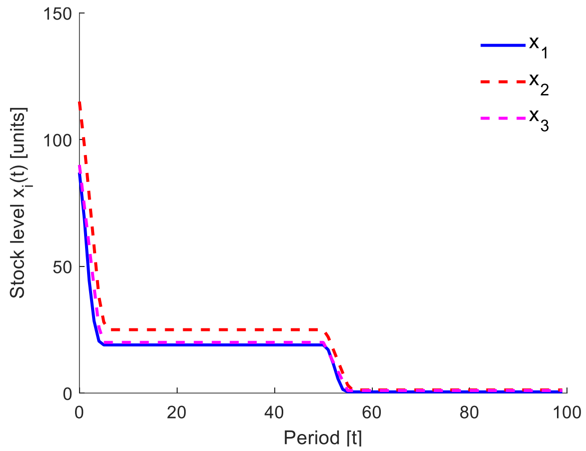

Fig. 17. The stock level at nodes 1–3, nominal case, Network A.

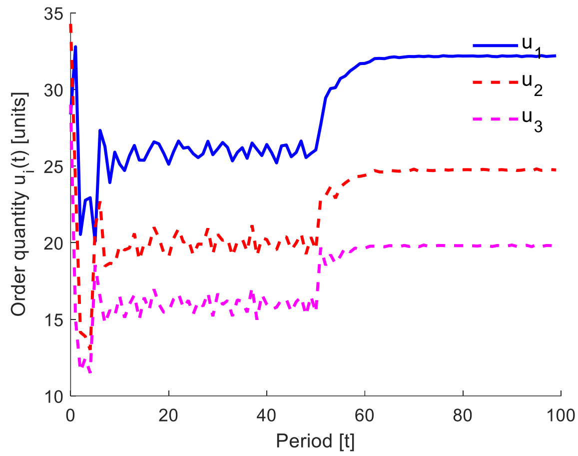

Fig. 18. Order quantity generated at nodes 1–3, under improper records with reduced stock level, Network A.

TABLE V. VARIANCE OF SYSTEM VARIABLES IN NETWORK A (NOMINAL NETWORKED CASE)

Parameter	**Measured value**			**Set**
node *i*	**1**	**2**	**3**	
var[d_i]	3.1	6.6	12.5	**A**
var[u_i]	8.6	9.7	12.5	**A**
var[x_i]	31.4	47.7	61.8	**A**
b_i	2.73	0.64	0.56	**A**
γ	1			**A**
γ^{max}	2.52			**A**
var[d_i]	0.9	1.7	3.1	**B**
var[u_i]	2.4	2.5	3.2	**B**
var[x_i]	8.7	13.1	15.3	**B**
b_i	2.61	0.62	0.55	**B**
γ	1			**B**
γ^{max}	2.57			**B**
var[d_i]	1.9	4.1	7.2	**C**
var[u_i]	5.3	6	7.3	**C**
var[x_i]	20.5	32.2	38.2	**C**
b_i	2.85	0.64	0.55	**C**
γ	1			**C**
γ^{max}	2.59			**C**

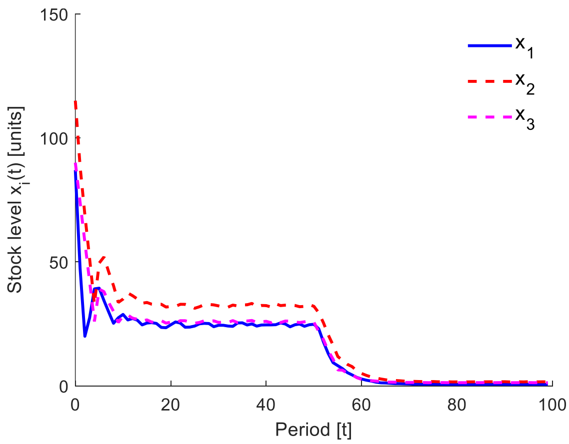

Fig. 19. Stock level at nodes 1–3, under improper records with reduced stock level, Network A.

2) Basic Network Configuration – Imprecise Stock Level Records

In the first assessed scenario, the stock level was reduced to 75–80% of the original value. Therefore, as depicted in Fig. 18, the order quantity starts to fluctuate, and hence not all the demands are fulfilled. Interestingly, as detailed in Table VI, the introduced networked BI indicator γ has diminished notably below 1 in cases **A** and **C** ($\gamma_{\mathbf{A}}$ = 0.86, $\gamma_{\mathbf{C}}$ = 0.66) by accordingly 14% for set **A** and even 34% in set **C**. Surprisingly, in set **B,** the BI has risen significantly by 34% to the value $\gamma_{\mathbf{B}}$ = 1.34. The demand variations have been throttled in the distribution network, yet the dampening intensity strongly correlates with the type of distribution. The most significant variance reduction was observed for the Poisson distribution. The stock level (Fig. 19) reaches a level above the expected minimum level, influencing higher holding costs in relation to the nominal case.

TABLE VI. VARIANCE OF SYSTEM VARIABLES IN NETWORK A (STOCK REDUCED)

Parameter	**Measured value**			**Set**
node *i*	**1**	**2**	**3**	
var[d_i]	3.2	6.6	12.1	**A**
var[u_i]	6.1	9	10.5	**A**
var[x_i]	29.2	47	64.5	**A**
b_i	1.9	0.7	0.5	**A**
γ	0.86			**A**
γ^{max}	2.11			**A**
var[d_i]	0.9	1.7	3.2	**B**
var[u_i]	2.8	4.2	4.3	**B**
var[x_i]	10.2	14.4	18.1	**B**
b_i	3.11	0.93	0.58	**B**
γ	1.34			**B**
γ^{max}	3.55			**B**
var[d_i]	1.9	4.1	7.2	**C**
var[u_i]	2.7	3.7	4.7	**C**
var[x_i]	15.9	28.1	40.1	**C**
b_i	1.41	0.54	0.44	**C**
γ	0.66			**C**
γ^{max}	1.55			**C**

An noteworthy discovery has been made in the opposite case when the stock level value used in the calculations was increased to 120–125% with respect to its original level. Contrary to the earlier findings, these results indicate that the networked BI increases significantly at all nodes in sets **A**, **B**, and **C**, ($\gamma_{\mathbf{A}}$ = 1.75, $\gamma_{\mathbf{B}}$ = 2.05, $\gamma_{\mathbf{C}}$ = 1.64), by accordingly 75%, 105%, and 64%, as specified in Table VII. The order quantity fluctuates significantly (Fig. 21), together with the stock level (Fig. 22), which leads to an unsatisfied demand (Fig. 20). Since γ > 1, the system does not preserve stability in the logistic sense.

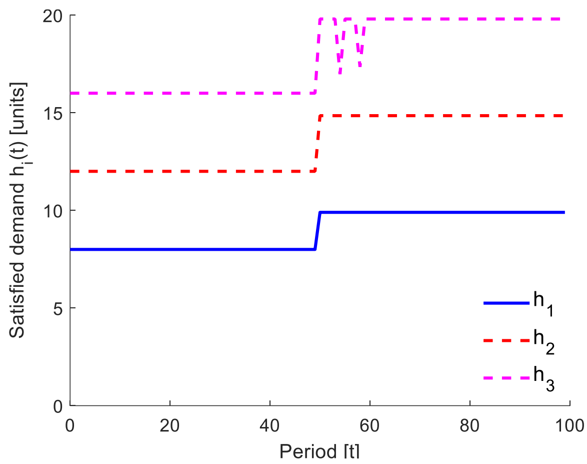

Fig. 20. Satisfied demand at nodes 1–3, under improper records with increased stock level, Network A.

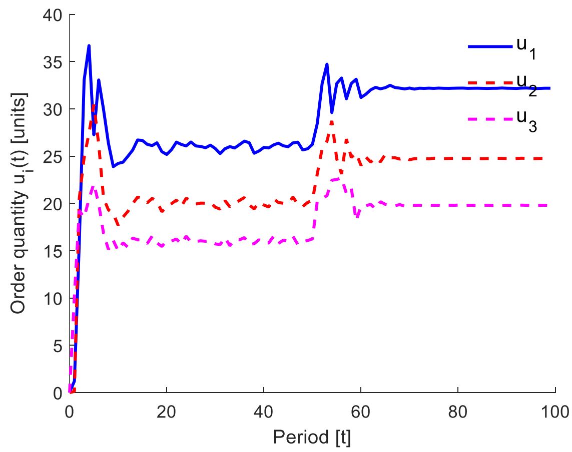

Fig. 21. Order quantity generated at nodes 1–3, under improper records with increased stock level, Network A.

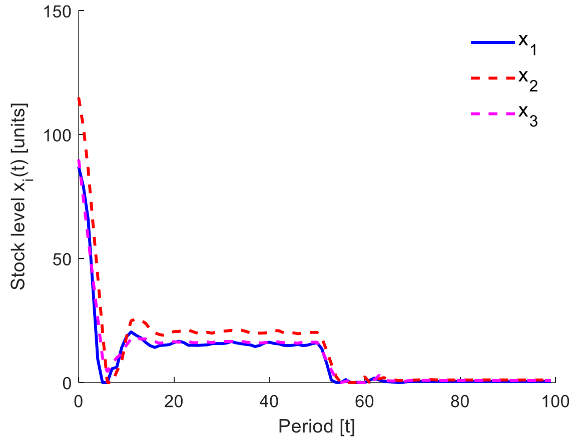

Fig. 22. The stock level at nodes 1–3, under improper records with increased stock level, Network A.

TABLE VII. VARIANCE OF SYSTEM VARIABLES IN NETWORK A (STOCK INCREASED)

Parameter	**Measured value**			**Set**
node i	**1**	**2**	**3**	
var[d_i]	3.1	6.6	12.1	**A**
var[u_i]	21.2	20.2	20	**A**
var[x_i]	55.8	66.7	65.2	**A**
b_i	6.75	0.73	0.62	**A**
γ	1.75			**A**
γ^{max}	5.08			**A**
var[d_i]	0.9	1.7	3.2	**B**
var[u_i]	6.5	6.2	5.9	**B**
var[x_i]	16.5	18.6	17.7	**B**
b_i	7.17	0.76	0.63	**B**
γ	2.05			**B**
γ^{max}	5.89			**B**
var[d_i]	1.9	4.1	7.2	**C**
var[u_i]	11.9	11.8	11.5	**C**
var[x_i]	31.5	39.2	37.4	**C**
b_i	6.31	0.74	0.6	**C**
γ	1.64			**C**
γ^{max}	4.87			**C**

3) *Real-World Network*

Afterwards, to reflect the circumstances exposing the current logistic industry shipment models, a real-world goods distribution network is considered. The arrangement examined in this section of the numerical study describes the actual European distribution network in the premium-clothes fashion industry. The enterprise root warehouses are in Paris and Milan, marked with orange circles. The distribution centers are located in Brussels, Munich, Berlin, Praga, Graz, Warsaw, Cracow, and Budapest, as shown in Fig. 24. The controlled nodes in the are marked with blue circles. The system graph representation is presented in Fig. 23. The numbers over the arrows are the channel utilization rate β_{ij} and lead times Λ_{ij} associated with the transportation links.

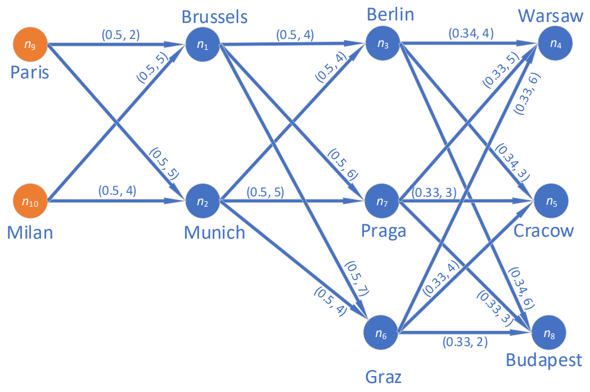

Fig. 23. Graph representation of real-world Network B. The arrows indicate the goods flow direction. The numbers denote β_{ij} and Λ_{ij}.

The logistic stability was evaluated quantitatively by examining BI (2) for each controlled node, the BE value in the networked context (14), and the BE maximum limit for the whole goods distribution network (15). The measured variance of d_i, u_i, and x_i at nodes 1–8 in response to demand signal given in Table I, is provided in Table VIII. The situation of supplying the external source is avoided by limiting dimension to $(N + S, N)$.

The performed simulations confirm that relations (5) and (7) do not hold anymore in the real-world distribution network Network B, depicted in Fig. 24. In the nominal operating conditions, i.e., when the stock records are properly registered, it could be interpreted that the system does not preserve the stability in a logistic sense, basing on the introduced networked indicators. Inside evaluated sets **A**, **B**, and **C,** the networked BI $\gamma > 1$ ($\gamma_{\mathbf{A}} = 1.46$, $\gamma_{\mathbf{B}} = 1.46$, $\gamma_{\mathbf{C}} = 1.5$). Therefore, from the perspective of applied networked indicators, one can conclude that the system is unstable. Hence, the more elaborated networked topology, having the number of connections per controlled node ($\delta = 2.87$) and the total number of nodes ($N + S = 10$), needs additional treatment to mitigate the BE. A more appropriate channel assignment strategy seems to be a place for further investigation.

TABLE VIII. VARIANCE OF SYSTEM VARIABLES IN NETWORK B (NOMINAL NETWORKED CASE)

Parameter	**Measured value**								**Set**
node *i*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
var[d_i]	3.2	6.7	12	8.7	2.6	5.1	7.8	6	**A**
var[u_i]	13.8	17.5	13.9	8.7	2.6	7	9.7	6	**A**
var[x_i]	54.2	95.8	71.6	47	10.9	40.0	62	22	**A**
b_i	4.28	0.87	0.47	0.39	0.23	0.94	0.67	0.39	**A**
γ	1.46								**A**
γ^{max}	6.58								**A**
var[d_i]	0.9	1.7	3.2	2.3	0.6	1.4	2.2	1.5	**B**
var[u_i]	3.9	4.7	3.7	2.3	0.6	1.9	2.5	1.6	**B**
var[x_i]	17.5	30.3	20.1	12.5	2.6	12	18.4	6.1	**B**
b_i	4.24	0.84	0.47	0.39	0.22	0.95	0.64	0.38	**B**
γ	1.46								**B**
γ^{max}	6.62								**B**
var[d_i]	1.9	4	7.2	5.3	1.5	3.1	4.6	3.6	**C**
var[u_i]	8.7	10.9	8.5	5.3	1.5	4.3	5.8	3.6	**C**
var[x_i]	45.9	81.5	46.8	30.8	6.3	28.9	47.6	14.8	**C**
b_i	4.61	0.85	0.47	0.39	0.22	0.95	0.65	0.38	**C**
γ	1.5								**C**
γ^{max}	6.73								**C**

The correlation of imprecise stock level records, with diminished stock level to 75–80% of the original value, is surprising with relation to the measured BE in the networked environment. Inside evaluated sets **A**, and **B,** the quantified BI $\gamma > 1$ ($\gamma_{\mathbf{A}} = 2.32$, $\gamma_{\mathbf{B}} = 5.73$), as grouped in Table IX. The significant enlargement of the BI is opposite behavior to the serial connection, wherein the stock reduced case revealed the BE avoidance. Here, in the real-world transportation network, imprecise stock level records, even with reduced stock level, caused huge BE amplification in the system. However, noteworthy, inside set **C**, the BE has been throttled in the system, having $\gamma < 1$ ($\gamma_{\mathbf{C}} = 0.82$). This study appears to be an essential insight to highlight how significant the importance of used stochastic distribution and its input parameters is to simulate the customer demand in relation to the BE appearance. Hence, it strongly impacts judging about the logistic stability with demand signal uncertainties and lead-times variations.

TABLE IX. VARIANCE OF SYSTEM VARIABLES IN NETWORK B (STOCK REDUCED CASE)

Parameter	**Measured value**								**Set**
node i	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
var[d_i]	3.2	6.8	11.9	8.9	2.5	5.2	7.8	6	**A**
var[u_i]	16.9	27.5	14.1	8	1.9	8.8	14.5	4.8	**A**
var[χ_i]	57.4	99.9	70.5	51.8	11.4	41.5	70.9	26.4	**A**
b_i	5.33	1.19	0.36	0.36	0.18	1.26	0.87	0.23	**A**
γ	2.32								**A**
γ^{max}	8.14								**A**
var[d_i]	0.9	1.7	3.1	2.3	0.6	1.4	2	1.5	**B**
var[u_i]	10.8	18	7.1	3.4	0.7	5.2	8.6	1.8	**B**
var[χ_i]	24.8	39.9	22.6	13.7	2.9	12.7	19.9	7.3	**B**
b_i	11.9	1.45	0.33	0.37	0.17	2.47	1.16	0.18	**B**
γ	5.73								**B**
γ^{max}	17.69								**B**
var[d_i]	1.9	4.1	7.4	5.2	1.5	3.1	4.7	3.6	**C**
var[u_i]	4.2	6.1	5.6	3.3	0.9	2.7	3.9	2.3	**C**
var[χ_i]	35.2	64.6	47.8	30.5	6.6	28	46.9	16.3	**C**
b_i	2.24	0.73	0.41	0.32	0.2	0.66	0.53	0.31	**C**
γ	0.82								**C**
γ^{max}	3.92								**C**

TABLE X. VARIANCE OF SYSTEM VARIABLES IN NETWORK B (STOCK INCREASED CASE)

Parameter	**Measured value**								**Set**
node i	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
var[d_i]	3.2	6.6	12	8.8	2.5	5.1	7.8	6	**A**
var[u_i]	42.8	51.4	25.7	14.5	4	14.3	20	9.4	**A**
var[χ_i]	103	191	82.9	51.1	11.2	48	75.4	21.2	**A**
b_i	13.3	1.04	0.41	0.42	0.23	1.59	0.91	0.36	**A**
γ	4.27								**A**
γ^{max}	15.13								**A**
var[d_i]	0.9	1.7	3.1	2.3	0.6	1.4	2	1.5	**B**
var[u_i]	15.7	19	8.2	4.3	1.1	5	7.2	2.7	**B**
var[χ_i]	40.2	72.8	25	13.8	2.8	15.9	24.2	5.9	**B**
b_i	17	1.09	0.37	0.41	0.22	2.02	1.01	0.31	**B**
γ	6.06								**B**
γ^{max}	20.14								**B**
var[d_i]	1.9	4.1	7.2	5.3	1.5	3.1	4.6	3.6	**C**
var[u_i]	23.3	27.4	14.6	8.3	2.3	7.9	10.2	5.5	**C**
var[χ_i]	69.9	129	53.8	30.6	6.3	34	50.9	13.6	**C**
b_i	12.4	1	0.42	0.42	0.23	1.46	0.83	0.4	**C**
γ	3.83								**C**
γ^{max}	13.9								**C**

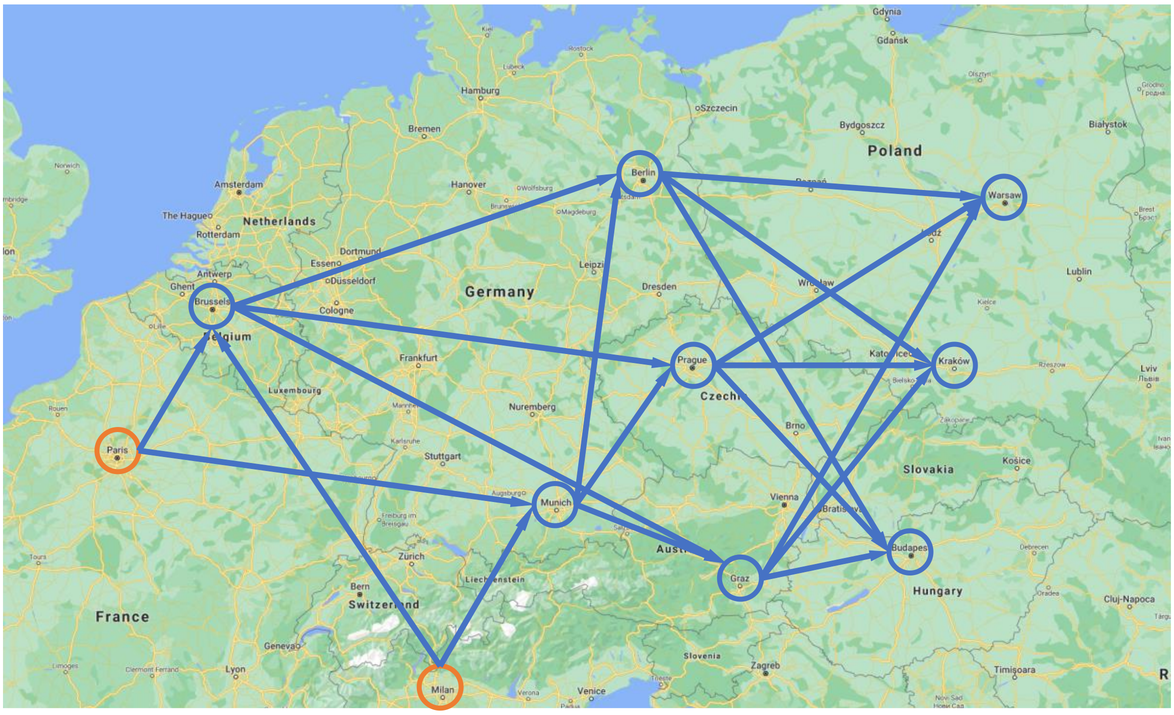

Fig. 24. Real-world goods distribution Network B.

An interesting finding is related to an opposite case study when the stock level value used in the calculations was enlarged to 120–125%, about its original level in inventory. Similar to Network A findings, these results suggest that the BI increases significantly at all nodes in sets **A**, **B**, and **C**, as stipulated in Table X. Since $\gamma > 1$ for all the assessed cases, the system does not keep stability in the logistic sense. Further optimization tactics are needed to be implemented to avoid system disturbances in the complex, networked goods distribution center.

VI. CONCLUSIONS

The paper introduces formal ground for the analysis of supply chain stability – in the *logistic* system perspective – in the presence of demand uncertainty and delay. Chain structures, organized in a serial connection with multiple demand points, were analyzed formally and numerically, and networked structures numerically. The OUT inventory policy is applied autonomously at each node to generate a suitable stock replenishment signal. For networked system, additional BI indicators are introduced to judge about the logistic stability in a network environment.

The order-to-demand variance ratio has been chosen as a principal measure of logistic stability, following a typical practice in the subject literature. It has been found that the

OUT policy does not trigger the BE, despite non-negligible time delay in the goods delivery, neither uncertain, time-varying demand, in the nominal operating conditions. The situation changes drastically when the system has a complex networked topology, which leads to a loss of logistics stability.

However, when the stock records are estimated imprecisely or contain errors originating from internal or external factors, the BE does appear in the system. In the logistic sense, the stability is not preserved both for serial and networked connections. The BI intensity depends on the type of demand distribution, with the most significant fluctuation increase reported for normal distribution. In addition to the loss of stability of goods flow, the excessive transportation appears in the system, leading to the increased logistic costs. The situation is particularly problematic when the records indicate bigger stock than that actually kept in the warehouse. In such a case, robust control methods are needed. The networked systems might also need channel allocation enhancements.

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