Modeling of probable maximum values in autonomous driving

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***Abstract*—In this paper, we approximate the probable maximum (very rare, extremal) values of highly autonomous driving sensor signals by reviewing two methods based on dynamic time series scaling and multifractal statistics.**

The article is a significantly revised and modified version of the conference material ("Determination of extreme values in autonomous driving based on multifractals and dynamic scaling") presented at the conference "2021 IEEE 15th International Symposium on Applied Computational Intelligence and Informatics, SACI".

The method of dynamic scaling is originally derived from statistical physics and approximates the critical interface phenomena. The time series of the vibration signal of the corner radar can be considered as a fractal surface and grow appropriately for a given scale-inverse dynamic equation. In the second method we initiate, that multifractal statistics can be useful in searching for statistical analog time series that have a similar multifractal spectrum as the original sensor time series.

Keywords—Highly autonomous driving (HAD), extreme value, probable maximum value, testing, dynamic scaling, multifractal analysis

I. INTRODUCTION

Highly autonomous driving (HAD) is expected to will have a positive impact on the global transport environment. According to the state of research, more than 80% of traffic accidents are caused by human error. By replacing the human operation with a technical-mathematical solution, it is possible to reduce the number of accidents.

Driver-free vehicles allow, for example, the reinterpretation of taxi services and the modernization of logistics.

Nowadays, the development of autonomous vehicles has been in full swing for years. OEMs (car manufacturers) promise to develop vehicles with a higher degree of autonomy in the coming years.

The driver of an autonomous vehicle can be deactivated, it cannot be taken into account by the operator of the control operations. As a result, there will be very high reliability requirements for safety, reliability and security.

For a practical interpretation and implementation of the safety requirements for self-driving vehicles, it is necessary to understand what reliable and safe behavior really means. For example, a HAD car must be able to handle traffic rules, the geometry and topology of its surroundings, and must be able to interpret the meteorological system, but also rare, difficult-to-predict road hazards. A strategy must then be devised to check that the vehicle has reached actually the required level of safety. The problem in the development of HAD in the coming years is the release of the systems: how the completed hardware-software and algorithm can be put on the market and how to sell the system.

The models used to determine and derive the extreme values used to test the HAD system are inherently based on various technical interactions, supplemented by the probability of possible collision damage. According to the current state of science, stochastic, linear operators are used to estimate the probability of very rare traffic incidents and the possible additional damage resulting from the failure of the detection system of the traffic environment [1,2].

Safety studies used in the design of the HAD system have shown that some test results based on existing autonomous driving test environments do not meet the extreme sensor values (PES) realized based on stochastic models [3,4]. PES is an extreme value for a given sensor environment that may occur in the future (8000 hours of autonomous driving). Several methods are known in the literature for HAD design to determine this value. These methods can be divided into two main systems: analytical and stochastic (Monte-Carlo) [1].

According to the current state of science and technique, methods need to be developed for estimating the probability of extreme sensor values that allow the for OEMs development of a probability-based sensor event standard that would lead to a methodology that allows a generalized definition of risk and cost [10]. HAD research projects have derived that existing methods are not suitable for predicting extreme sensor values.

The reason for the discrepancy may be that the methods used to derive the extreme values of the sensor time series result from standard linear statistical techniques that cannot be used when the object or phenomenon to be detected is essentially nonlinear.

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One previous analysis of the dynamic characteristics of time series of autonomous sensor signals was performed by Bianchi et al. [6], through the study of the chaotic attractor analysis [7,8]. The dimension of the attractor algebra found was low and not an integer value: fractal, indicating that the system can be described with high probability by nonlinear functions of some variables that also apply to the extreme values of the sensor time series. The fact that the correlation dimension found was fractal predicts that this function, also called an attractor in nonlinear dynamics [2], may show chaotic behavior under

given initial and circumferential conditions. The existence of a low-dimensional mapping of attractor algebra may suggest the extreme-value prediction, which means short-term prediction possibilities in time series mathematics using some nonlinear model [3-8].

In deep learning technology is regularly used to predict short-term, extreme values of chaotic sensor time series mapping a nonlinear model [9]. We obtained good sensor event predictions using probabilistic neural networks [21, 22], and in a previous work we did not support the hypothesis that random fluctuations in the time series of sensor values are caused by linear Gaussian noise [23, 24]

This article analyzes the extremes of time series of corner radar sensors used in the HAD system with the aim of comparing two different nonlinear techniques:

- dynamic scaling and
- multifractal formalism.

Dynamic scaling is a procedure developed in statistical physics for the characterization and quantitative analysis of physical interfaces. In general, interfaces can be formed in three processes:

- interfaces grow with the addition of new substances,
- substances that detach or dissolve from the surface or interface,
- and finally processes that lead to spontaneous processes.

In the present project, in which we present the tests applied in the Autonomous Driving System of the University of Óbuda, the corner radar sensor time series (Figures 1 and 2) plays the role of the interface, and the second type of process (materials removed from the surface or interface) prevails. Due to the importance of HAD time series analysis, it is a challenge to characterize the extremal value of time series and the dynamics of such extremes, and to develop an efficient approach to embedded systems to understand the formation of time series. Recently, significant progress has been made in understanding interfaces through the application of fractal concepts and the development of the theory of dynamic scaling. The dynamic scaling introduced in fractal growth has become an indispensable tool for the characterization and fractal-based, nonlinear analysis of the morphology and evolution of interfaces and can also be applied in the theoretical study of surfaces.

A very important step in identifying the extremal values of the HAD corner radar sensor sequence would be to examine the correlation associated with fluctuations in sensor extremes. The Hurst exponent (H) and the Hölder exponent (α) are widely used to measure the persistence of a statistical phenomenon [18]. H <0.5 points is an anti-persistent time series. It characterizes a system that can also be used in a corner radar system that reverses more often and travels less distance than the components of a random system. $0.5 < H <1$ means that a persistent time series is analyzed. This system has significant memory

effects in the study of extremes: what happens now affects the future, so there is a very deep dependence on initial conditions. Persistent processes are common in nature, but they also play an important role in self-driving [19]. If the distribution is homogeneous, then there is a unique $\alpha = H$, but if not, there are multiple α exponents. The most common α will characterize the series and will play as a Hurst exponent. A very efficient method for determining the $f(\alpha)$ spectra of autonomous corner radar sensors is based on the multifractal analysis tool introduced in the early eighties.

Theoretical evidence gathered in recent years shows that the time series of the corner radar sensor, which is preferred for autonomous driving, have multifractal scaling properties such as high variability, intermittency, and multiscaling [7,8,9,10]. These studies suggest that multifractal scaling of sensor time series can be a valuable tool for characterizing extremal sensor value, i.e., searching for statistically physically similar time series that are much more "extremal" but have the same multifractal spectrum.

In this journal article, we review the basic ideas of dynamic scaling and multifractal analysis, as well as how to characterize the maximum value of autonomous radar sensor time series. To illustrate the application of these ideas, we then use data from 13593 autonomous radar sensors in the HAD database for a scenario (Figure 2).

In the Chapter II we are dealing with fractal dynamic scaling, with the first-order consideration of radar extremities. The methods for multifractal analysis are described in Chapter III. This is followed in Chapter IV by the first results on corner radar, and in Chapter V by a summary and summary of future work.

II. Fractal dynamic scaling

Few indicators of HAD deal with time series and their nonlinear properties in many systems.

In certain applications, the goal is to produce a time series with a specific physical or technical property, but often time series are inherent in industrial and natural processes. In fact, Mandelbrot [15] pointed out that some time series are best approximated by a fractal geometry system. This recognition led to the development of a dynamic scaling system that describes not only a given morphology but also the internal dynamics of time series, including extremes. In this approach, we consider the time evolution of time series in a d-dimensional space, starting from an initial corner radar time series.

The essence of the method is the physical experience that growing surface-instabilities with the same scaling factor are physically identical, i.e. they can be scaled together.

The increase of the instabilities is realized by analyzing the morphology of the time series. As a first step, the active zone of the time series, or the part of the time series responsible for the fluctuations, is determined.

In our case, the time series of the vertical vibrations of the corner radar signal can be described by the function *Q(r,t),* which gives the autonomous radar sensor at position r and at time *t*.

Figure 1. The corner radar sensor ultimately measures distance, but taking the parameters shown at the top of the figure into account is an essential part of sensor detection accuracy. In our Óbuda University Autonomous Driving research project, we look for possible extreme values of vertical vibration values of the corner radar issues.

Let the average height of the time series at time *t* be [1]:

$$Q^{-} = \langle Q(r,t)\rangle = \frac{\sum_r Q(r,t)}{L^{d-1}} \tag{1}$$

where denotes the average over *r*.

The measurement time series of the sensors fluctuates around this average value, and the root mean square value of these fluctuations *w(L,t)* is a quantitative measure of the width of the time series and is as follows:

$$w(L,t) = \sqrt{[\langle Q^2(r,t)\rangle - \langle Q(r,t)\rangle^2]} \tag{2}$$

The roughness, β of the time-series can be defined as:

$$w(L,t) \approx t^{\beta} \tag{3}$$

that is the exponent β describes the time-series fluctuations growth in time [1].

The maximum spatial measure is the length of the time series by which these fluctuations can increase in the d-1 dimensions.

For seeking the extremum, the corner radar measures has uncertainties, which means that an additional signal, the vibration time series must also be taken into account.

Based on the vertical vibration, the extremity of the system can be inferred, because as soon as the fluctuations in the sensor values reach this length, they can no longer increase, and the "surfaces" of the time series reach a steady state, characterized by a constant latitude value. From the fractality, we can assume that the surface of the time series is scale-varying, and the dynamic saturation value of the width achieved over a long period of time is expected to be power-law dependent in L2 :

$$w(L, t \to \infty) \approx L^{\alpha} \tag{4}$$

with the characteristic exponent α [6].

In a persistent, steady state, the vertical vibration time series surface is best described by self-affine and fractal geometry. According to the geometry, the morphology is quantified with α, which is equivalent to the Hurst exponent *H*. The dependence of *w(L,t)* on *t* and *L* can be combined with Equations (3) and (4) into a single expression representing the dynamic scaling

$$wL,t) = L^{\alpha} f\left(\frac{t}{L^{\frac{\propto}{\beta}}}\right) \tag{5}$$

where the scaling function $f(x) \approx x^{\beta}$ for $x \ll 1$ [7].

Going further in the analysis, the analyzed width of the vertical vibration sensor time series is independent of *t* and saturates to a constant value, taking into account the metrics of fractal geometry. At this limit, *w(L,t)* varies according to Equation (4), and the scaling function *f (x)* is a constant. The dynamic scaling identity of Equation (5) means that the representations of *w(L,t)* for different *L* values as a function Equation (5) fall on a single curve.

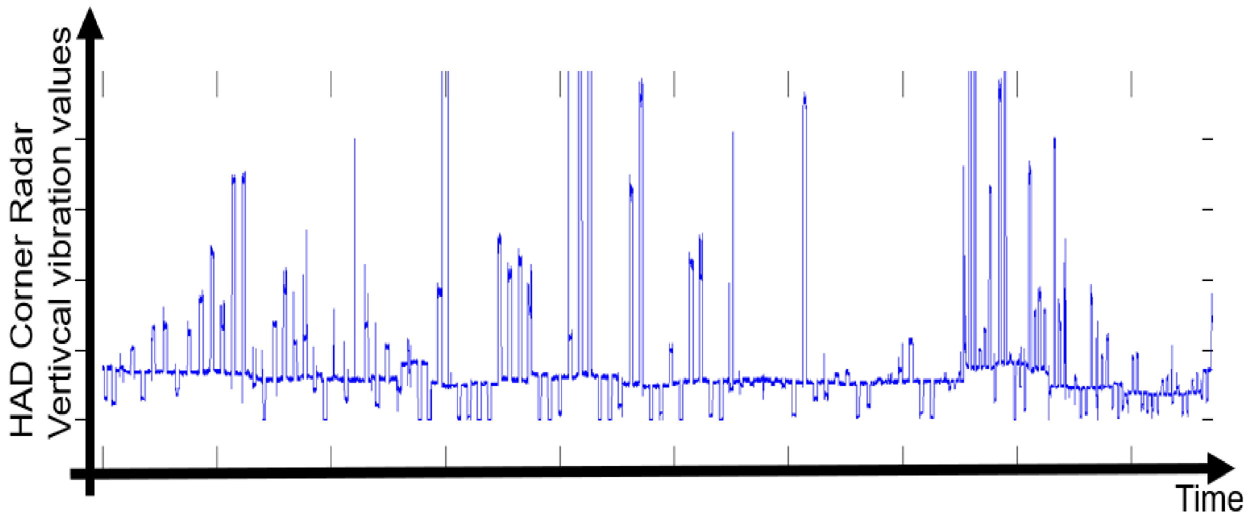

Figure 2. Vertical vibration of the corner radar time series. The radar sensor ultimately measures distance, but taking parameters shown in Figure 1 into account is an essential part of sensor detection accuracy.

The extremum value based on the dynamic scaling behavior for sensor time series has been observed in models with different dimensions and in several experiments [8], and the extremum can be determined based on the α, β coefficients.

Consider, for example, the height difference correlation function c(r,t) defined by

$$c(r,t) = \langle |Q^{-}(r',t') - Q^{-}(r+r',t+t')|^2 \rangle \qquad (6)$$

where *Q(r,t)* is the time series of the radar sensor vertical vibration, responsible for changing the probability of a vehicle collision.

On the basis of the dynamic scaling form (5), for $r \ll L$ the correlation functions *c(r,0)* is expected to have the following scaling form:

$$c(r,0) \approx r^{2\alpha} for\ r \ll L \qquad (7)$$

and for fixed scales as:

$$c(r,0) \approx r^{2\beta} for\ t \ll \ \tau \qquad (8)$$

The scaling behaviors of Eqs.(7)-(8) persist as long as *r* is smaller than the length *L* and *t* is smaller than $t \approx L^{\frac{\alpha}{\beta}}$

Within these limits the correlation functions

$$c(r \to \infty, 0) \text{ and } c(0, t \to \infty)$$

saturate to constant value that depend on *L*. [9,10].

The correlation diagram shows the strength of each element of the vertical vibration time series. The diagram was realized by recording the correlation of a large number of 128-element ring buffers realized from the original time series.

Figure 3 shows the correlation diagram c(r,t) of the HAD vertical vibration signal. The dynamic scaling behavior of the vibrations characteristic of the autonomous sensor time series and the exponents α and β allow the determination of the extremal (probable maximum) sensor value. [12]. From the figure, which is shown in the form of a scalogram, the time coordinate of the extreme value is simply conspicuous. In this case, it is called the extremum, which takes an extreme value with the correlations.

It can be seen from the figure that based on the first tests, no significant difference can be found in the correlation diagram, they can be easily scaled into each other. The essence of the test was to take into account 4 participants and 12 participants (objects to be detected, which also play a role in vertical vibration). and considering the entropy-changing effect of the increased information content in the correlation.

III. Multifractal Analysis

The main features of self-similar fractal objects, such as time series of vibration signal of corner radar measurement results, are their scaling properties related to magnification invariance. For uniform fractals, a scaling exponent called the fractal dimension uniquely describes the scaling. However, most traffic node dynamics occurring fractal objects have multifractal properties. The traffic node participants can be more fully characterized by the spectrum of fractal exponents *D(q)*, where *q* is a real number, the so-called generalized dimension, where the fractal dimension is equal to *D(0)*, is usually called the multifractal spectrum. [13].

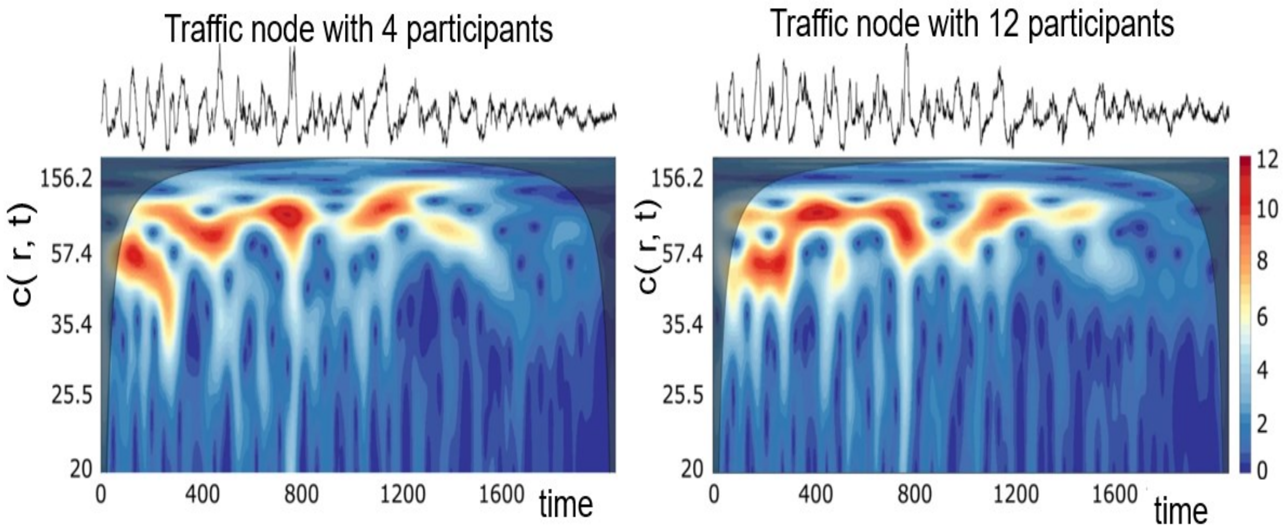

Figure 3. Óbuda University Autonomous Driving Software Radar correlation function *c(r,t)* for 4 and 12 traffic node participants, the difference is not conspicuous

The general goal of multifractal formalism is to determine the singularity spectrum *f(α)* of a μ measure [14,15]. The Haussdorff dimension of each point [15] is associated with the singularity exponent α, which gives an idea of the strength of the singularity.

$$N_\alpha(\varepsilon) \approx \varepsilon^{f(\alpha)} \quad (9)$$

where $N_\alpha(\varepsilon)$ the number of boxes needed to cover the measure and ε is the size of each box [16].

A partition function *Z* can be defined from this spectrum (it is the same model as the thermodynamic one).

$$Z(q,\varepsilon) = \sum_{\iota=1}^{N(\uparrow\varepsilon)} \mu_\iota^q(\varepsilon) \approx \epsilon^{\tau(q)} \text{ for } \epsilon \to 0 \quad (10)$$

where τ(*q*) is a spectrum which arouses by Legendre transforming the *f* (α) singularity spectrum. The spectrum of generalized fractal dimensions: $D_;$ is obtained from the spectrum τ(*q*)

$$D_q = \frac{\tau(q)}{q-1} \quad (11)$$

The capacity or box dimension of the support of the distribution is given by

$$D_0 = f\big(\alpha(0)\big) = -\tau(0)$$

$$D_1 = f\big(\alpha(1)\big) = \alpha(1)$$

It is proportional to the scale of information dissemination and is called the information dimension. For *q>2*, the correlation integrals D_q and *q*-point are related.

A widely used method for calculating the multifractal spectrum can be done with q generally between q_{min} and q_{max} in 0.2 steps. In this range, the error bands of the multifractal spectra of the corner radar time series values used in autonomous guidance developments are quite small (see Figure 3 below), so this *q* range is suitable for characterizing the time series of autonomous sensors with multifractal exponents.

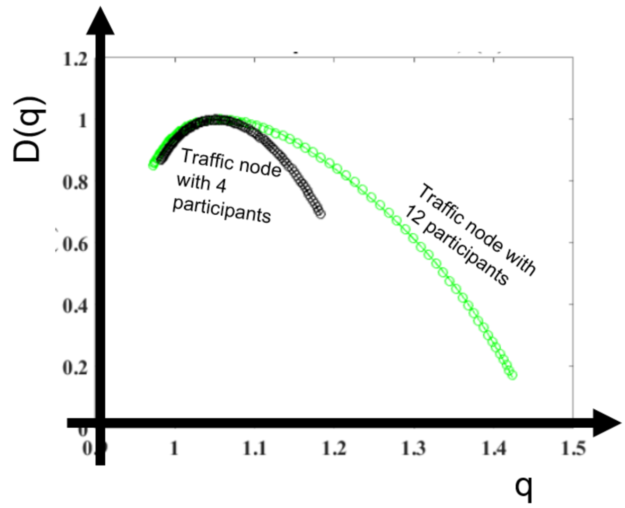

Figure 4. Óbuda University Autonomous Driving Software Radar multifractal spectra of the radar sensor vertical vibration, green traffic node with 12 participants, black 4 participants. Analysis with 12 participants is more widespread with a multifractal property, indicating a greater uncertainty effect on detection than.

IV. METHOD, FIRST NUMERICAL RESULTS

The embedded version practical procedure for calculating extreme values for example for the HAD radar experiments could be seen of the Figure 5.

- Collecting 128 HAD vertical vibration corner radar values into a *R* ringbuffer;
- In the microcontroller unit, determining the multifractal spectra of *R* based on the α and β parameters.
- Generating *N* samples from the α, β measure (*N*= 1024, depends on the microcontroller environment)
- Determining the supremum of the signal amplitude of the signals
- The supremum of the possible maximum value of a HAD corner radar signal for the measure *w* and samples *N*

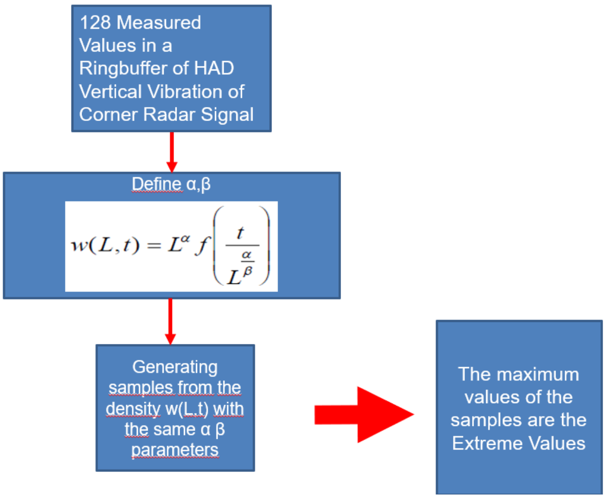

Figure 5. Óbuda University Autonomous Driving Software Radar analysis: extreme value prediction flow chart

With help of dynamic scaling if r=124 (ms) then $c(124,t)$ = 82 and then when t→ ∞ is and $\alpha = \frac{1}{2}\frac{\lg(124)}{\lg(82)} = 0.5269$ → then the extreme value

$$w \approx 14763^{0.5269} = 157.4 \text{ [Radar\_unit]}$$

With help of multifractal analysis, looking for the similar time series own similar spectra, the extremum value is

$$w = 129.1 \text{ [Radar\_unit]}.$$

Determination of extreme values in autonomous driving based on multifractals and dynamic scaling, however this iteration method is very time consuming.

V. Conclusions, Future work

Within the framework of the Self-Driving Automotive Platform Project running at the University of Óbuda, we analyzed nonlinear methods for predicting the probable maximums, the so called extremes of autonomous driving vibration signal of the corner radar sensor time series.

The first conclusions

- Determining the extreme value of a radar signal can be significantly simplified by nonlinear time series analysis, thereby making it embedded suitable.
- The need for testing can be reduced if collisions can be predicted by analyzing the extreme values of the radar sensor parameters.
- New test results can be integrated into the system constantly.

The next phase of the project is the fixed-point Matlab C++ Code Generation and its testing with real data [17-20].

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