Global Position Feedback Tracking Control of a Serial Robot Manipulator with Revolute Joints

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***Abstract*—In this paper, we present the controller which globally stabilizes a non-stationary motion of a serial robot manipulator with revolute joints without velocity measurements. A family of desired manipulator motions is considered such that the first vertical link of the manipulator performs a given rotation, and the remaining links retain the given relative angular positions. It is proved that such motions of the manipulator can be made globally asymptotically stable using dynamic position feedback. The problem is solved taking into account the periodicity of the dynamics equations along the angular coordinates of the links. As an example, a numerical simulation of the three-link manipulator motion under the constructed controller is presented.**

***Index Terms*—stabilization control problem, serial robot manipulator, revolute joint, dynamic position feedback, Lyapunov function, cylindrical phase space**

I. Introduction

In control theory, trajectory tracking is a fundamental problem. Trajectory tracking of a multi-link robot manipulators is considered as challenging control problem due to nonlinearity and non-stationarity of the dynamics equations. The main approach to the solution of the trajectory tracking control problem for a serial robot manipulator is the construction of proportional derivative (PD) controller with feedforward [1], [2]. Note that the use of a PD controller requires the position and velocity measurements of the manipulator links. In practice, the use of tachometers is fraught with difficulties. This is, firstly, the noise of the signals of the measured speeds, and secondly, the installation of tachometers makes the robot heavier and increases its cost. In addition, in some practically important tasks, for example the installation of tachometers is impossible when the robot operates in an aggressive environment, in a hot cell, etc.

The majority of work for control design of robotic manipulators without velocity measurements uses the dynamic filters, see [2]–[5]. For results related to the use of velocity observers see [6], [7] and for nonlinear proportional integral controllers and Volterra integro-differential equations see [8]–[12]. Due to the complexity of the problem, results on the global trajectory tracking of robot manipulators without velocity measurements are scarce. Note that the problem on global output trajectory tracking control of Euler-Lagrange systems has been solved in [3] based on Lyapunov function method.

Motivated by the authors' early works for the trajectory tracking control problem of multi-link robot manipulators [13], [14], in this paper, we give the solution to the global trajectory tracking control problem without velocity measurements for the revolute joined robotic arms with a vertical first link. The key contributions of our paper can be written as follows:

1) We use the periodicity property of the robotic manipulators equipped with revolute joints. Due to this property, we construct the dynamic position feedback controller which is bounded in position term and ensures the global attractivity of the reference trajectory in a cylindrical phase space.

2) We ensure the global tracking of reference trajectories such that the first link rotation angle is unbounded and twice continuously differentiable function with both derivatives bounded, and other link rotation angles are constant.

Throughout this paper, the following notation is used. Symbol $|\cdot|$ indicates the vector norm in $\mathbb{R}^n$. Symbol $\|\cdot\|$ denotes the operator matrix norm corresponding to the vector norm $|\cdot|$. $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ denote the smallest and largest eigenvalues of some matrix respectively. Symbol $\mathcal{K}$ denotes the Hanh functions class.

The paper is organized as follows. In Section II we present the mathematical model of a robotic arm and define the problem setting. Our main result is stated in Section III. Example of a motion control for a three-link robot manipulator that illustrates our main results is presented in Section IV. Conclusions are provided in Section V.

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II. Mathematical Model of a Robotic Arm and Problem Formulation

We consider serial robotic arms described by Euler-Lagrange equations such as

$$A(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u, \quad (1)$$

where $q \in \mathbb{R}^n$, $\dot{q} \in \mathbb{R}^n$, and $\ddot{q} \in \mathbb{R}^n$ are the vectors of joint rotation angles, angular velocities, and angular accelerations respectively, $A(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the matrix of Coriolis and centrifugal terms, $g(q) \in \mathbb{R}^n$ represents the gravitational torques, $u \in \mathbb{R}^n$ is the vector of control torques.

Consider some properties of the robotic arms (1).

1. The matrix $A(q)$ satisfies the following inequalities

$$\|A(q)\| \le d_1 \quad \forall q \in \mathbb{R}^n,$$
$$d_2 \le \lambda_{min}(A(q)) \le \lambda_{max}(A(q)) \le d_3 \quad \forall q \in \mathbb{R}^n,$$

where d_1, d_2, and d_3 are some positive reals.

2. The following holds

$$\dot{A}(q(t)) = C(q(t),\dot{q}(t)) + C^T(q(t),\dot{q}(t))$$
$$\forall q \in \mathbb{C}^1,\ q:\ [0,+\infty) \to \mathbb{R}^n.$$

3. $\forall e_1, e_2 \in \mathbb{R}^n$, the Coriolis matrix $C(e_1,e_2)$ satisfies the following inequalities

$$\lambda_{max}(C(e_1,e_2) + C^T(e_1,e_2)) \le \lambda_{c1}\|e_2\|,$$
$$\|C(e_1,e_2)\| \le \lambda_{c2}\|e_2\|,$$

where $\lambda_{c1} > 0$ and $\lambda_{c2} > 0$ are some constants.

The focus of our paper is on the following properties of the revolute joined robotic arms with a vertical first link.

4. The inertia matrix $A(q)$ and potential energy $\Pi(q)$ of the manipulator do not depend on the first link rotation angle. This angle is said to be a cyclic coordinate and the rotation angles of the other links are said to be positional ones.

5. The matrices $A(q)$, $C(q,\dot{q})$ and the vector $g(q)$ in (1) are periodic functions of the variables $q_1, q_2, \ldots, q_n$ with some periods $h_i > 0$ ($i = 1,2,\ldots,n$) respectively. So, if $u = 0$, then, the system (1) has not only one equilibrium position $(q,\dot{q}) = (0,0)$ but a whole set of equilibrium positions $(q,\dot{q})$ such as $q = (h_1k_1, h_2k_2, \ldots, h_nk_n)^T$, $\dot{q} = 0$, where $k_j \in \mathbb{Z}$, $j = 1,2,\ldots,n$.

For the mechanical system (1) assume that the output vector contains only link positions. Let find a position feedback controller u which moves the manipulator (1) from any initial position with any initial velocity to track a desired trajectory. Let us mathematically formulate the control problem.

Define the set Q of all desired trajectories of (1) such as

$$\begin{gathered} Q = \{q_r(t):\ [t_0,+\infty) \to \mathbb{R}^n: \\ \|\dot{q}_{r1}(t)\| \le q_{m1}, \quad \|\ddot{q}_{r1}(t)\| \le q_{m2}, \\ q_{ri} = constant, \quad i = 2,3,\ldots,n\}, \end{gathered} \quad (2)$$

where $q_{r1}(t)$ is a twice differentiable function, $q_{mi} = constant > 0$ $(i = 1,2)$, $t_0 = constant \ge 0$.

The problem consists in constructing a controller $u = u(t, q(t), q(t+s))$ $(s \in [-t, 0])$ such that the desired trajectory $q_r(t) \in Q$ of the manipulator (1) is uniformly asymptotically stable and globally attractive.

III. Robotic Arm Trajectory Tracking

Choose some desired trajectory $q_r(t) \in Q$ and denote the tracking errors as follows

$$e_q = q - q_r(t), \quad \dot{e}_q = \dot{q} - \dot{q}_r(t). \quad (3)$$

From (1) one can obtain the error dynamic equations

$$A_{st}(e_q)\ddot{e}_q + C_{st}(e_q, 2\dot{q}_r(t) + \dot{e}_q)\dot{e}_q = u - u_r(t,e_q), \quad (4)$$

where

$$\begin{gathered} A_{st}(e_q) = A(e_q + q_r(t)), \\ C_{st}(e_q,x) = C(e_q + q_r(t), x), \\ u_r(t,e_q) = A(q_r(t) + e_q)\ddot{q}_r(t) \\ +C(q_r(t) + e_q, \dot{q}_r(t))\dot{q}_r(t) + g(q_r(t) + e_q). \end{gathered} \quad (5)$$

Let us introduce a cylindrical phase space for (4) such as

$$\{(e_q, \dot{e}_q) \in \mathbb{K}^n \times \mathbb{R}^n\},$$

where $\mathbb{K}^n$ is given by

$$\mathbb{K}^n = \{x \in \mathbb{R}^n:\ x_1(\mathrm{mod} h_1), x_2(\mathrm{mod} h_2), \ldots, x_n(\mathrm{mod} h_n)\}.$$

Consider the controller u such as follows

$$\begin{gathered} u = u_r(t,e_q) + u_{st}(e_q,x), \\ u_{st}(e_q,x) = -K_p p(e_q) - K_x x, \\ \dot{x} = -a(x + b\dot{e}_q), \end{gathered} \quad (6)$$

where $a = constant > 0$ and $b = constant > 0$, $K_p, K_x \in \mathbb{R}^{n\times n}$ are some gain constant matrices, $x = x(t, t_0, e_{q0}, \dot{e}_{q0}, x_0)$ is a solution of a differential equation from (6), $p = p(e_q)$, $p:\ \mathbb{R}^n \to \mathbb{R}^n$ is a continuously differentiable function such that $p(0) = 0$ and $p(e_q) = (p_1(e_{q1}), p_2(e_{q2}), \ldots, p_n(e_{qn}))^T$.

Using the integration by parts formula one can obtain the solution $x(t,t_0,x_0)$ of an ordinary differential equation in (6) with position measurements only. So, one can obtain

$$\begin{gathered} x(t,t_0,e_{q0},\dot{e}_{q0},x_0) = x_0e^{-a(t-t_0)} - ab(e_q(t) \\ -e^{-a(t-t_0)}e_{q0} + a^2b\int\limits_{t_0}^{t} e_q(s)e^{-a(t-s)}ds). \end{gathered} \quad (7)$$

Using (4) and (6), one can easily obtain the closed-loop system such as

$$\begin{gathered} A_{st}(e_q)\ddot{e}_q + C_{st}(e_q, 2\dot{q}_r(t) + \dot{e}_q)\dot{e}_q \\ +K_p p(e_q) + K_x x = 0, \\ \dot{x} = -a(x + b\dot{e}_q). \end{gathered} \quad (8)$$

Note that using (7) one can obtain the first equation of (8) is functional differential [15].

The equilibrium positions of (8) are contained in the set

$$P = \{(e_q,\dot{e}_q,x) \in \mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n:\ p(e_q) = 0,\ \dot{e}_q = 0,\ x = 0\}. \quad (9)$$

Define the subset of (9) as follows

$$\begin{gathered} S = \{(e_q,\dot{e}_q,x) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n: \\ s(e_q) = 0,\ \dot{e}_q = 0,\ x = 0\}. \end{gathered} \quad (10)$$

where $s(e_q) = (s_1(e_{q1}), s_2(e_{q2}), \ldots, s_n(e_{qn}))^T$, $s_j(e_{qj}) = \int\limits_0^{e_{qj}} p_j(z)dz$, $j = \overline{1,n}$.

Consider the following definitions of global attractivity, uniform stability, and uniform asymptotic stability properties of the sets (9) and (10).

Definition 1. *The solution set (9) of the closed-loop system (8) is said to be globally attractive if* $(\forall \varepsilon > 0)(\forall t_0 \geq 0)$ $(\forall (e_{q0}, \dot{e}_{q0}, x_0)^T \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n)$ $(\exists \sigma > 0)$ $(\forall t \geq t_0 + \sigma)$ $\|(p(e_q(t, t_0, e_{q0}, \dot{e}_{q0}, x_0)), \dot{e}_q(t, t_0, e_{q0}, \dot{e}_{q0}, x_0), x(t, t_0, e_{q0}, \dot{e}_{q0}, x_0))^T\| < \varepsilon$.

Definition 2. *The solution set (10) of the closed-loop system (8) is said to be uniformly stable if* $(\forall \varepsilon > 0)$ $(\exists \delta = \delta(\varepsilon) > 0)$ $(\forall t_0 \geq 0)$ $(\forall (e_{q0}, \dot{e}_{q0}, y_0) \in \{(x, \dot{e}_q, y) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$: $\|(p(e_q), \dot{e}_q, x)^T\| < \delta$ $)$ $(\forall t \geq t_0)$ $\|(s(e_q(t, t_0, e_{q0}, \dot{e}_{q0}, x_0)), \dot{e}_q(t, t_0, e_{q0}, \dot{e}_{q0}, x_0), x(t, t_0, e_{q0}, \dot{e}_{q0}, x_0))^T\| < \varepsilon$.

Definition 3. *The solution set (10) of the closed-loop system (8) is said to be uniformly asymptotically stable if it is uniformly stable and uniformly attractive. The uniform attractivity property seems that* $(\exists \Delta > 0)$ $(\forall \varepsilon > 0)$ $(\exists \sigma > 0)$ $(\forall t_0 \geq 0)$ $(\forall (e_{q0}, \dot{e}_{q0}, x_0) \in \{(e_q, \dot{e}_q, x) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$: $\|(p(e_q), \dot{e}_q, x)^T\| < \Delta$ $)$ $(\forall t \geq \sigma + t_0)$ $\|(s(e_q(t, t_0, e_{q0}, \dot{e}_{q0}, x_0)), \dot{e}_q(t, t_0, e_{q0}, \dot{e}_{q0}, x_0), x(t, t_0, e_{q0}, \dot{e}_{q0}, x_0))^T\| < \varepsilon$.

The following theorem presents the main contribution of this paper.

Theorem 1. *Let the controller (6) be such as*

$$K_p = wE, \quad K_x = -abE, \tag{11}$$

where w, a and b are some positive constants.

Then, the solution set P of the closed-loop system (8) is globally attractive and the solution set S is uniformly asymptotically stable.

Proof.

Consider the Lyapunov function candidate $V = V(e_q, \dot{e}_q, x)$ such as follows

$$V = \tfrac{1}{2}(\dot{e}_q)^T A_{st}(e_q)\dot{e}_q + w\sum_{i=1}^{n} s_i(e_{qi}) + \tfrac{1}{2}x^T x. \tag{12}$$

Note that $V(e_q, \dot{e}_q, x) \geq 0 \; \forall (t, e_q, \dot{e}_q, x) \in \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$. Moreover, there exists a function $\omega_1 \in \mathcal{K}$ such that

$$V(e_q, \dot{e}_q, x) \geq \omega_1(||(s(e_q), \dot{e}_q, x)^T||). \tag{13}$$

The time derivative of the Lyapunov function candidate V is calculated as

$$\begin{aligned} \dot{V} &= \tfrac{1}{2}(\dot{e}_q)^T \dot{A}_{st}(e_q)\dot{e}_q + (\dot{e}_q)^T A_{st}(e_q)\,(\ddot{e}_q) \\ &\quad + w(p(e_q))^T\dot{e}_q + x^T\dot{x} \\ &= \tfrac{1}{2}(\dot{e}_q)^T \dot{A}_{st}(e_q)(\dot{e}_q) + (\dot{e}_q)^T \\ &\quad \times(-C_{st}(e_q, 2\dot{q}_r(t) + \dot{e}_q)\dot{e}_q - K_p p(e_q) - K_x x) \\ &\quad + w(p(e_q))^T\dot{e}_q - ax^T x - ab\dot{e}_q^T x. \end{aligned} \tag{14}$$

From (14), one can obtain

$$\begin{aligned} \dot{V} &= (\dot{e}_q)^T(C_{st}(e_q, -\dot{q}_r(t))\dot{e}_q \\ &\quad + p^T(e_q)(wE - K_p)\dot{e}_q \\ &\quad + x^T(-abE - K_x)\dot{e}_q - ax^T x. \end{aligned} \tag{15}$$

In can easily see that $(\dot{e}_q)^T(C_{st}(e_q, -\dot{q}_r(t))\dot{e}_q = 0$. Then, from (15) using (11), one can get the following inequality

$$\dot{V} = -ax^T x \leq 0. \tag{16}$$

The set $\{\dot{V} = 0\}$ consists of the solutions of (8) such that $\{x = 0\}$. So, from (8) one can see that such solutions satisfy the following

$$\dot{e}_q = 0, \quad p(e_q) = 0. \tag{17}$$

Thus, one can conclude that the solution set (9) of (8) is globally attractive.

Note now that the function $V = V(e_q, \dot{e}_q, x)$ satisfies the inequalities

$$\omega_1(||(s(e_q), \dot{e}_q, x)^T||) \leq V \leq \omega_2(||(s(e_q), \dot{e}_q, x)^T||), \tag{18}$$

where $\omega_1, \omega_2 \in \mathcal{K}$.

Then, using (18) and Lyapunov stability theory, one can obtain that the solution set (10) is uniformly asymptotically stable. This completes the proof.

Note that the coefficients in (11) can be chosen as any positive constants, their value affects the rate of convergence of the real motion of the manipulator to the desired one.

Note also that the global trajectory control problem for robotic manipulators has been solved in [3]. The main differences between our result and the known one [3] are as follows. 1. In our paper, unbounded time functions can be chosen as reference trajectories. 2. In our paper, the problem has been solved in a cylindrical phase space, which made it possible to use a bounded proportional term in the controller. 3. The conditions of Theorem 1 do not coincide with ones from [3], and these conditions are not a special case of ones from [3].

IV. Global Tracking of a 3-DOF Robotic Manipulator

Consider the performance of the controller (6) for a 3-DOF robotic arm like as PUMA-560 (see, Fig. 1).

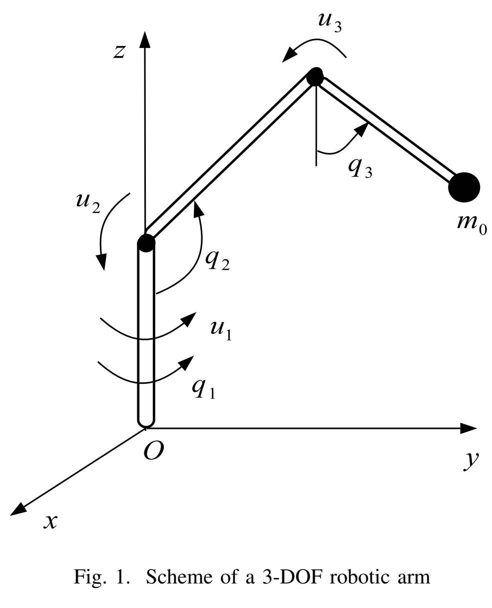

Fig. 1. Scheme of a 3-DOF robotic arm

Assume that the generalized coordinates $q_1 = \varphi_1$, $q_2 = \varphi_2$, and $q_3 = \varphi_3$ are the angular displacements of the cylindrical joints O_1, O_2, and O_3 respectively. The dynamics of a 3-DOF serial robot manipulator with cylindrical joints is defined by (1).

The components a_{ij} of $A(q)$ are given by:

$$a_{11} = I_1 + m_2 l_{c2}^2 \sin^2(q_2) + m_4(l_2 \sin(q_2) + l_{c3} \sin(q_3))^2,$$
$$a_{12} = a_{13} = a_{21} = a_{31} = 0,\ a_{22} = m_2 l_{c2}^2 + m_3 l_2^2,$$
$$a_{23} = a_{32} = m_4 l_2 l_{c3} \cos(q_2 - q_3)/2,\ a_{33} = m_4 l_{c3}^2.$$

where l_2 is the length of the second link; m_j is the mass of the link j; m_0 is the mass of a load; $m_4 = m_0 + m_3$; I_1 is the inertia moment of the first link with respect to Oz; l_{c2} and l_{c3} are the lengths of the intervals between the mass centers of the second link and the third one with a load and the rotation axes of these links accordingly.

The components c_{ij} of $C(q, \dot{q})$ are given by:

$$c_{11} = (m_2 l_{c2}^2 + m_4 l_2^2) \sin(2q_2)\dot{q}_2/2$$
$$+ m_4 l_2 l_{c3}(\sin(q_2)\cos(q_3)\dot{q}_3 + \cos(q_2)\sin(q_3)\dot{q}_2)$$
$$+ m_4 l_{c3}^2 \sin(2q_3)\dot{q}_3/2,$$
$$c_{12} = -c_{21} = (m_2 l_{c2}^2 + m_4 l_2^2)\sin(2q_2)\dot{q}_1/2$$
$$+ m_4 l_2 l_{c3} \sin(q_3)\cos(q_2)\dot{q}_1,$$
$$c_{13} = -c_{31} = m_4 l_2 l_{c3} \sin(q_2)\cos(q_3)\dot{q}_1$$
$$+ m_4 l_{c3}^2 \sin(2q_3)\dot{q}_1/2,$$
$$c_{22} = c_{33} = 0,\ c_{23} = m_4 l_2 l_{c3} \sin(q_2 - q_3)\dot{q}_3/2,$$
$$c_{32} = -m_4 l_2 l_{c3} \sin(q_2 - q_3)\dot{q}_2/2.$$

The components $g_j,\ j = 1, 2, 3$ of the vector $g(q)$ are as follows:

$$g_1 = 0,\ g_2 = (m_2 l_{c2} + m_4 l_2) g \sin q_2,$$
$$g_3 = m_4 l_{c3} g \sin q_3.$$

The functions $p_j : \mathbb{R} \to \mathbb{R},\ j = \overline{1,3}$ are given by

$$p_j(e_{qj}) = \sin\left(\frac{e_{qj}}{2}\right),\ j = \overline{1,3}.$$

The functions $s_j : \mathbb{R} \to \mathbb{R},\ j = \overline{1,3}$ are given by

$$s_i(e_{qj}) = 2\left(1 - \cos\left(\frac{e_{qj}}{2}\right)\right),\ j = \overline{1,3}.$$

One can easily see that the sets P and S can be written as

$$P = \{(e_q, \dot{e}_q, x) \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 :$$
$$e_{qj} = 2\pi k_j\ (j = \overline{1,3}),\ k_j \in \mathbb{Z},\ \dot{e}_q = 0,\ x = 0\},$$
$$S = \{(e_q, \dot{e}_q, x) \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 :$$
$$e_{qj} = 4\pi k_j\ (j = \overline{1,3}),\ k_j \in \mathbb{Z},\ \dot{e}_q = 0,\ x = 0\}.$$

The robot parameters are given as

$$I_1 = 0.1\, kg \cdot m^2,$$
$$m_2 = 13.8\, kg,\quad m_3 = 4.9\, kg,\quad m_0 = 3.1\, kg,$$
$$l_2 = 1.6\, m,\quad l_{c2} = 0.7\, m,\quad r_3 = 0.5\, m.$$

The desired trajectory is chosen as

$$q_{1r}(t) = (3t)\, rad,\ q_{2r} = \pi/2\, rad,\ q_{3r} = \pi/4\, rad.$$

The controller is given by (6). The parameters of (6) are chosen such as

$$a = 10,\ b = 1,\ K_p = 2E,\ K_x = -10E. \tag{19}$$

Let the initial state and velocity of the manipulator be such as

$$q_1(0) = 3.0 + q_{1r}(0)\, rad,\ q_2(0) = -2.0 + q_{2r}\, rad\ ,$$
$$q_3(0) = 2.1 + q_{3r}\, rad,$$
$$\dot{q}_1(0) = 40\, rad/s,\ \dot{q}_2(0) = -35\, rad/s,$$
$$\dot{q}_3(0) = 50\, rad/s.$$

The simulation has been performed using Scilab 5.5.2 platform.

Figs. 2 – 4 show the tracking process for the desired trajectory. One can easily see the asymptotic convergence of the links trajectories to the desired ones plus $4\pi z$, where $z = (z_1, z_2, z_3)^T,\ z_j \in \mathbb{Z},\ j = 1, 2, 3$.

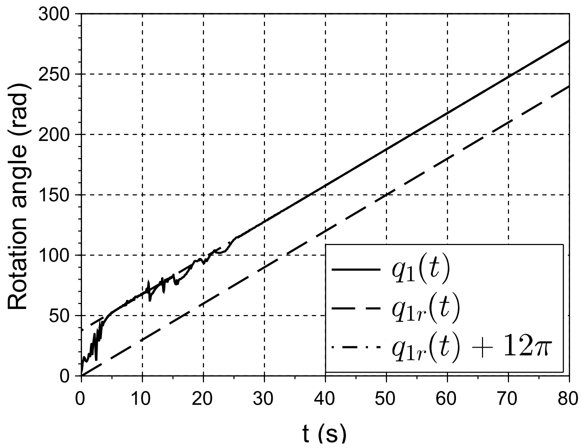

Fig. 2. Desired and actual angular coordinate for the first joint.

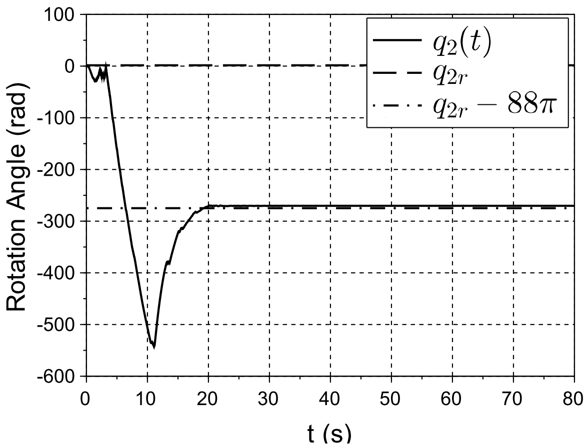

Fig. 3. Desired and actual angular coordinate for the second joint.

In Fig. 5 the time evolution of the stabilizing control torques has been shown. Thus, it can be seen from Figs. 2 – 4 that the solution to the global trajectory tracking control problem is obtained.

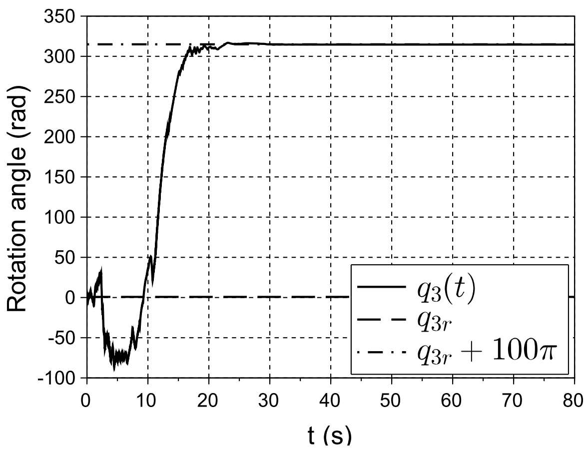

Fig. 4. Desired and actual angular coordinate for the third joint.

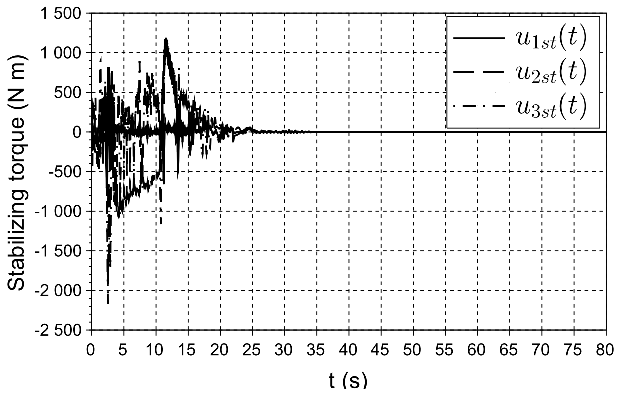

Fig. 5. Stabilizing control torques.

V. Conclusion

In this paper we have presented results that justify the design of a dynamic position feedback controller based on Lyapunov function method for a robotic arm trajectory tracking without velocity measurements. The first-order dynamic filter has been designed in order to compensate the absence of velocity measurements. We have shown that the controller with arbitrary small gain matrices provides the uniform asymptotic stability and global attractivity properties for the reference trajectories of a serial robot manipulator with revolute joints such that the first link rotates around vertical line and other links hold constant relative positions. It has been proved that the global trajectory tracking holds in a cylindrical phase space. In other words, from any initial state at any initial velocity, each link of the manipulator tends asymptotically to the motion displaced by a multiple of 2π from a desired one. The values of the gain matrices affect the rate of the real motion convergence to the given one of the manipulator. The theoretical results that we have presented for a multi-link robot manipulator have been demonstrated in numerical simulation of a three-link robotic arm like as PUMA-560.

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