Limiting State Estimation of Switched Interval Systems with Metzler-Takagi-Sugeno Models

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***Abstract*—The paper deals with interval observer design for fuzzy switched positive systems. The systems are represented by the Takagi-Sugeno fuzzy models, with premise variables depending on a measurable part of the state vector. Stability conditions for the proposed interval observer structure are formulated via linear matrix inequalities to ensure nonnegative system state estimation. The proposed method allows to compute the lower and upper bounds of the system state under the assumption that the system disturbance are bounded. The properties of proposed approach are illustrated in numerical example.**

***Index Terms*—Takagi-Sugeno models, switched systems, Metzler systems, parametric constraints, interval observer design.**

I. Introduction

The state observers of nonlinear dynamical systems has always been a challenging research topic with large field of applications. Useful for practical purposes have been shown the Takagi-Sugeno (T-S) fuzzy models [1], addressing linear models local dynamics to describe sector-bounded nonlinear systems and preferring the state-space representation of systems. Consequently, the developments in this field [2], [3] have traversed closely paralleled to linear system control theory, where the key schemes contribute to maintain feasibility of the linear matrix inequalities (LMI) [4].

Confining attention to the problems utilizing the same features for systems with nonnegative states [5], [6], the concept linearizing the equations describing the system is conditioned by additional system parametric constraints [27] and the theory of Metzler matrices [8], [9], to reflect the system positiveness. An suitable unification is the LMI-based design strategy for positive Metzler systems, proposed in [10], reflecting the diagonal stabilization principle of systems.

Counterpart to systems with known and fixed matrix parametric representation, the interval approach is outlined in [11], [12] to provide the bounded system state estimation for given system matrix bounds. In addition, [13] presents an approach for interval observer analysis using Metzler matrix properties and these conditions motivated the interval observers design approach for T-S systems in [14], [15]. The switched systems introduces another principal difficulty that follows from desired system state positivity [16].

The same principal questions arising above are combined in the case of control design for T-S fuzzy switched systems [17], [18], [19], [20], [21] and in the technique of interval observers for switched T-S fuzzy systems [22], where positiveness becomes more relevant. It has to be underlined that pervasive deployments of these approaches have constituted an important base platform in controller and interval observer design for uncertain switched T-S fuzzy systems.

The challenge, of course, is to extended the above standalone solutions associated with T-S fuzzy systems in designing the interval observers for Metzler-Takagi-Sugeno (M-T-S) fuzzy switching systems. Formulating problem with LMI based preference, the interval switching observer stability conditions reflect standard arguments and incorporate the notion of diagonal stabilization that has to be in keeping when working with the Metzler system matrix structures. The results presented in this paper substantially extend and strengthen the results given in [23] to accomplished the relationships between system matrix parametric constraints, the LMIs feasibility and the observer state upper and lover vector state estimation. Because the only tools from the field of LMIs complexity are effectively deployed defining the switched interval fuzzy observer conditions, practical aspects are standard.

The paper is organized as follows. In Section II the observer design for T-S fuzzy systems is adduced. For given class of M-T-S fuzzy switched systems the set of LMIs, describing the design conditions for M-T-S fuzzy switched interval observer, is presented in Section III and the solution is illustrated Section IV by a numerical illustrative example. Within the underlying concept, Section V draws conclusions and some topics of the authors' research activity in the future.

Throughout the paper, the following notations are used: $\boldsymbol{x}^{\mathrm{T}}$, $\boldsymbol{X}^{\mathrm{T}}$ denotes the transpose of the vector $\boldsymbol{x}$, and the matrix $\boldsymbol{X}$, respectively, diag $[\cdot]$ marks a (block) diagonal matrix, for a square symmetric matrix $\boldsymbol{X} \prec 0$ means that $\boldsymbol{X}$ is negative definite matrix, the symbol $\boldsymbol{I}_n$ indicates the n-th order unit matrix, $\mathbb{R}$ ($\mathbb{R}_+$) qualifies the set of (nonnegative) real numbers, $\mathbb{R}^{n\times n}$ ($\mathbb{R}_+^{n\times n}$) refers to the set of (nonnegative) real matrices and $\mathbb{R}_{-+}^{n\times n}$ covers the set of Metzler matrices.

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II. Takagi-Sugeno Fuzzy Switching Observer

The used class of n-dimensional continuous-time switching systems are the sector nonlinear dynamic systems, represented by the T-S fuzzy compact form description

$$\dot{\boldsymbol{q}}(t) = \sum_{i=1}^{s} \mathrm{h}_i^\sigma(\boldsymbol{\vartheta}(t))(\boldsymbol{A}_i^\sigma \boldsymbol{q}(t) + \boldsymbol{B}_i^\sigma \boldsymbol{u}(t)) + \boldsymbol{D}\boldsymbol{d}(t) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}^\sigma \boldsymbol{q}(t) \tag{2}$$

actual where $\boldsymbol{q}(t) \in \mathbb{R}^n$, $\boldsymbol{u}(t) \in \mathbb{R}^r$, $\boldsymbol{y}(t) \in \mathbb{R}^m$, are vectors of the state, input, and output variables, $\boldsymbol{d}(t) \in \mathbb{R}^d$ is the bounded additive disturbance and $\boldsymbol{A}_i^\sigma \in \mathbb{R}^{n\times n}$, $\boldsymbol{B}_i^\sigma \in \mathbb{R}^{n\times r}$, $\boldsymbol{C}^\sigma \in \mathbb{R}^{m\times n}$, $\boldsymbol{D} \in \mathbb{R}^{n\times d}$ are the local model parameters.

Related to a switching sequence $\sigma(t)$ defined on $\mathbb{R}^+$ and taking values in the set $\Sigma = \{1, \dots n_s\}$, where n_s is a positive integer, the index σ denotes an active switching mode.

The T-S fuzzy model from a given nonlinear dynamical model can be obtained by the sector nonlinearity approach [4], whilst $\mathrm{h}_i^\sigma(\boldsymbol{\theta}(t))$ is averaging weight for the i-th fuzzy rule, representing the normalized grade of membership, where for all $i \in \{1, \dots, s\}$, $\sigma \in \{1, \dots, n_s\}$

$$0 \le \mathrm{h}_i^\sigma(\boldsymbol{\vartheta}(t)) \le 1, \quad \sum_{i=1}^{s} \mathrm{h}_i^\sigma(\boldsymbol{\vartheta}(t)) = 1 \tag{3}$$

while s is the number of the fuzzy rules and

$$\boldsymbol{\vartheta}(t) = \begin{bmatrix}\theta_1(t) \ \theta_2(t) \ \cdots \ \theta_o(t)\end{bmatrix} \tag{4}$$

is o-dimensional vector of premise variables. It is supposed in the following that the premise variables are composed only on the set of variables from $\boldsymbol{q}(t)$ and all are measurable, so $\boldsymbol{\vartheta}(t)$ is available online.

The task can be turned to construction of the T-S fuzzy observer, switching together with the system and defined as

$$\begin{aligned}\dot{\boldsymbol{q}}_e(t) = &\sum_{i=1}^{s} h_i^\sigma(\boldsymbol{\vartheta}(t))(\boldsymbol{A}_i^\sigma \boldsymbol{q}_e(t) + \boldsymbol{B}_i^\sigma \boldsymbol{u}(t)) + \\ &+ \sum_{i=1}^{s} h_i^\sigma(\boldsymbol{\vartheta}(t))\boldsymbol{J}_i^\sigma \boldsymbol{C}^\sigma(\boldsymbol{q}(t) - \boldsymbol{q}_e(t))\end{aligned} \tag{5}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}^\sigma \boldsymbol{q}(t) \tag{6}$$

where $\boldsymbol{J}_i^\sigma \in \mathbb{R}^{n\times m}$ for $i \in \{1, \dots, s\}$, $\sigma \in \{1, \dots, n_s\}$ are the observer parameters and $\boldsymbol{q}_e(t) \in \mathbb{R}^n$ is the observer state.

Introducing the errors in the observation

$$\boldsymbol{e}(t) = \boldsymbol{q}(t) - \boldsymbol{q}_e(t), \quad \boldsymbol{e}_y(t) = \boldsymbol{C}^\sigma \boldsymbol{e}(t) \tag{7}$$

then the dynamics of the state observation error $\boldsymbol{e}(t)$ is evolved according to the following equation

$$\begin{aligned}\dot{\boldsymbol{e}}(t) &= \sum_{i=1}^{s} h_i^\sigma(\boldsymbol{\vartheta}(t))(\boldsymbol{A}_i^\sigma - \boldsymbol{J}_i^\sigma \boldsymbol{C}^\sigma)\boldsymbol{e}(t) + \boldsymbol{d}(t) \\ &= \sum_{i=1}^{s} h_i^\sigma(\boldsymbol{\vartheta}(t))\boldsymbol{A}_{ei}^\sigma \boldsymbol{e}(t) + \boldsymbol{d}(t)\end{aligned} \tag{8}$$

where an observer switching parameter structure is

$$\boldsymbol{A}_{ei}^\sigma = \boldsymbol{A}_i^\sigma - \boldsymbol{J}_i^\sigma \boldsymbol{C}^\sigma \tag{9}$$

Theorem 1: If in the fuzzy model (1), (2) are included the mentioned assumptions on the premise variables and there exist a symmetric positive definite matrix $\boldsymbol{P} \in \mathbb{R}^{n\times n}$, the matrices $\boldsymbol{V}_i^\sigma \in \mathbb{R}^{n\times m}$ and a positive scalar $\xi \in \mathbb{R}_+$ such that for all $i \in \{1, \dots, s\}$, $\sigma \in \{1, \dots, n_s\}$ the following set of LMIs

$$\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} \succ 0, \quad \xi > 0 \tag{10}$$

$$\begin{bmatrix}\boldsymbol{P}\boldsymbol{A}_i^\sigma + \boldsymbol{A}_i^{\sigma\mathrm{T}}\boldsymbol{P} - \boldsymbol{V}_i^\sigma \boldsymbol{C}^\sigma - \boldsymbol{C}^{\sigma\mathrm{T}}\boldsymbol{V}_i^{\sigma\mathrm{T}} & * & * \\ \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P} & -\xi \boldsymbol{I}_d & * \\ \boldsymbol{C}^\sigma & \boldsymbol{0} & -\xi \boldsymbol{I}_m\end{bmatrix} \prec 0 \tag{11}$$

is feasible then, guaranteeing the attenuation of the disturbances effect by an upper bound ξ of the H_∞ norm of the disturbance transform matrix, the switching T-S fuzzy observer (5), (6), is asymptotically stable and the observer gains can be find for $i \in \{1, \dots, s\}$, $\sigma \in \{1, \dots, n_s\}$ as

$$\boldsymbol{J}_i^\sigma = \boldsymbol{P}^{-1}\boldsymbol{V}_i^\sigma \tag{12}$$

Proof: A positive Lyapunov function candidate $v(\boldsymbol{e}(t)$ can be served for a stable fuzzy equation (8) if with a symmetric positive definite matrix $\boldsymbol{P} \in \mathbb{R}^{n\times m}$ and with a positive scalar $\xi \in \mathbb{R}_+$

$$\begin{aligned}v(\boldsymbol{e}(t)) = &\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}(t) + \\ &+ \xi^{-1}\int_0^t (\boldsymbol{e}_y^{\mathrm{T}}(\tau)\boldsymbol{e}_y(\tau) - \xi^2 \boldsymbol{d}^{\mathrm{T}}(\tau)\boldsymbol{d}(\tau))\mathrm{d}\tau \\ &> 0\end{aligned} \tag{13}$$

Calculating the time derivative of $v(\boldsymbol{e}(t)$ along the trajectory of the fuzzy relation (8) the following outcome

$$\begin{aligned}\dot{v}(\boldsymbol{e}(t)) = &\dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}(t) + \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{e}}(t) + \\ &+ \xi^{-1}\boldsymbol{e}_y^{\mathrm{T}}(t)\boldsymbol{e}_y(t) - \xi \boldsymbol{d}^{\mathrm{T}}(t)\boldsymbol{d}(t)\end{aligned} \tag{14}$$

has to be prescribed as negative.

Thus, by substituting (8), it can be seen that

$$\begin{aligned}\dot{v}(\boldsymbol{e}(t)) = &\sum_{i=1}^{s} h_i^\sigma(\boldsymbol{\vartheta}(t))\boldsymbol{e}^{\mathrm{T}}(t)(\boldsymbol{A}_{ei}^{\sigma\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_{ei}^\sigma)\boldsymbol{e}(t) + \\ &+ \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{D}\boldsymbol{d}(t) + \boldsymbol{d}^{\mathrm{T}}(t)\boldsymbol{D}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e}(t)) + \\ &+ \xi^{-1}\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{C}^{\sigma\mathrm{T}}\boldsymbol{C}^\sigma \boldsymbol{e}(t) - \xi \boldsymbol{d}^{\mathrm{T}}(t)\boldsymbol{d}(t) \\ &< 0\end{aligned} \tag{15}$$

Letting the following

$$\boldsymbol{e}_d^{\mathrm{T}}(t) = \begin{bmatrix}\boldsymbol{e}^{\mathrm{T}}(t) \ \boldsymbol{d}^{\mathrm{T}}(t)\end{bmatrix} \tag{16}$$

it can be concluded that

$$\dot{v}(\boldsymbol{e}_d(t)) = \sum_{i=1}^{s} h_i^\sigma(\boldsymbol{\vartheta}(t))\boldsymbol{e}_d^{\mathrm{T}}(t)\, \boldsymbol{\Xi}_i^\sigma \boldsymbol{e}_d(t) < 0 \tag{17}$$

where, by the used labeling,

$$\boldsymbol{\Xi}_i^\sigma = \begin{bmatrix}\boldsymbol{A}_{ei}^{\sigma\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_{ei}^\sigma + \xi^{-1}\boldsymbol{C}^{\sigma\mathrm{T}}\boldsymbol{C}^\sigma & * \\ \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P} & -\xi \boldsymbol{I}_d\end{bmatrix} \prec 0 \tag{18}$$

Then, by Schur complement, it follows from (18) that

$$\begin{bmatrix} \boldsymbol{P}\boldsymbol{A}_{ei}^{\sigma}+\boldsymbol{A}_{ei}^{\sigma\mathrm{T}}\boldsymbol{P} & * & * \\ \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P} & -\xi\boldsymbol{I}_d & * \\ \boldsymbol{C}^{\sigma} & \boldsymbol{0} & -\xi\boldsymbol{I}_m \end{bmatrix} \prec 0 \tag{19}$$

By following the observer error system matrix structure, the design parameters can be included as

$$\boldsymbol{P}\boldsymbol{A}_{ei}^{\sigma}=\boldsymbol{P}\boldsymbol{A}_{i}^{\sigma}-\boldsymbol{P}\boldsymbol{J}_{i}^{\sigma}\boldsymbol{C}^{\sigma}=\boldsymbol{P}\boldsymbol{A}_{i}^{\sigma}-\boldsymbol{V}_{i}^{\sigma}\boldsymbol{C}^{\sigma} \tag{20}$$

where, for $i\in\{1,\dots,s\}$, $\sigma\in\{1,\dots,n_s\}$,

$$\boldsymbol{V}_{i}^{\sigma}=\boldsymbol{P}\boldsymbol{J}_{i}^{\sigma} \tag{21}$$

Thus, it can be concluded that (20) implies (11). The proof is complete. ∎

Remark 1: This is a convex optimization problem which can be solved in polynomial time [24] using an available LMI toolbox. If the above set of LMIs is feasible and the observer matrix gains are calculated as (12) for $i\in\{1,\dots,s\}$, $\sigma\in\{1,\dots,n_s\}$ then $\boldsymbol{q}_e(t)$ can be solved using a convex combination of fuzzy matrix parameters by (11).

It is no restriction in generality to assume that the vector of unknown disturbance $\boldsymbol{d}\in\mathbb{R}^d$ is performed through a matrix $\boldsymbol{D}\in\mathbb{R}^{n\times d}$.

III. Metzler-Takagi-Sugeno Fuzzy Switching Interval Observers

In this focus there are considered only known interval values of a nonnegative disturbance $\boldsymbol{d}(i)\in\mathbb{R}_{+}^{n}$, nonnegative matrices $\boldsymbol{B}_{i}^{\sigma}\in\mathbb{R}_{+}^{n\times r}$, $\boldsymbol{C}\in\mathbb{R}_{+}^{n\times n}$ and a strictly Metzler $\boldsymbol{A}_{i}^{\sigma}\in\mathbb{R}_{-+}^{n\times n}$, where the notation strictly Metzler $\boldsymbol{A}_{i}^{\sigma}$ means that all off diagonal elements of $\boldsymbol{A}_{i}^{\sigma}$ are greater then zero and all diagonal elements of $\boldsymbol{A}_{i}^{\sigma}$ are negative for all $i\in\{1,\dots,s\}$, $\sigma\in\{1,\dots,n_s\}$. This class of system parameter representations means an interval strictly M-T-S fuzzy switching positive system, characterized by nonnegative state vectors within the interval bounds. The interval constraints mean that for all $i\in\{1,\dots,s\}$, $\sigma\in\{1,\dots,n_s\}$ the system parameters, disturbance and the system state vector satisfy element-wise

$$\underline{\boldsymbol{A}}_{i}^{\sigma}\le\boldsymbol{A}_{i}^{\sigma}\le\overline{\boldsymbol{A}}_{i}^{\sigma},\quad \underline{\boldsymbol{C}}^{\sigma}\le\boldsymbol{C}^{\sigma}\le\overline{\boldsymbol{C}}^{\sigma} \tag{22}$$

$$\boldsymbol{0}\le\underline{\boldsymbol{q}}(0)\le\boldsymbol{q}(0)\le\overline{\boldsymbol{q}}(0),\quad \underline{\boldsymbol{\vartheta}}(t)\le\boldsymbol{\vartheta}(t)\le\overline{\boldsymbol{\vartheta}}(t) \tag{23}$$

and, moreover, for all $t\ge 0$

$$\underline{\boldsymbol{d}}\le\boldsymbol{d}(t)\le\overline{\boldsymbol{d}},\quad \underline{\boldsymbol{d}}=-\overline{\boldsymbol{d}} \tag{24}$$

Assumption (24) is standard in relation to the interval observers [25], where the uncertainties are assumed to be bounded with known bounds.

Since for a positive system (1), (2) the conditions (23) yield, the interval observer for this M-T-S switched structure is the generalization of (5) in such a way that for

$$\overline{\boldsymbol{y}}(t)=\overline{\boldsymbol{C}}^{\sigma}\boldsymbol{q}(t),\quad \underline{\boldsymbol{y}}(t)=\underline{\boldsymbol{C}}^{\sigma}\boldsymbol{q}(t) \tag{25}$$

it can be prescribed if the premise variables are measurable

$$\begin{aligned}\dot{\overline{\boldsymbol{q}}}_e(t)=&\sum_{i=1}^{s}h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))(\overline{\boldsymbol{A}}_{i}^{\sigma}\overline{\boldsymbol{q}}_e(t)+\boldsymbol{B}_{i}^{\sigma}\boldsymbol{u}(t))+\\ &+\sum_{i=1}^{s}h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))\boldsymbol{J}_{i}^{\sigma}\underline{\boldsymbol{C}}^{\sigma}(\boldsymbol{q}(t)-\overline{\boldsymbol{q}}_e(t))\end{aligned} \tag{26}$$

$$\begin{aligned}\dot{\underline{\boldsymbol{q}}}_e(t)=&\sum_{i=1}^{s}h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))(\underline{\boldsymbol{A}}_{i}^{\sigma}\underline{\boldsymbol{q}}_e(t)+\boldsymbol{B}_{i}^{\sigma}\boldsymbol{u}(t))+\\ &+\sum_{i=1}^{s}h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\boldsymbol{J}_{i}^{\sigma}\overline{\boldsymbol{C}}^{\sigma}(\boldsymbol{q}(t)-\underline{\boldsymbol{q}}_e(t))\end{aligned} \tag{27}$$

where the design objective constraints can be stated for a positive $t\ge 0$ as

$$\boldsymbol{0}\le\underline{\boldsymbol{q}}_e(t)\le\boldsymbol{q}(t)\le\overline{\boldsymbol{q}}_e(t) \tag{28}$$

if $\overline{\boldsymbol{q}}_e(0)=\overline{\boldsymbol{q}}(0)$, $\underline{\boldsymbol{q}}_e(0)=\underline{\boldsymbol{q}}(0)$.

Supposing only to the upper and lower M-T-S fuzzy limiting interval system description

$$\dot{\overline{\boldsymbol{q}}}(t)=\sum_{i=1}^{s}\mathrm{h}_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))(\overline{\boldsymbol{A}}_{i}^{\sigma}\overline{\boldsymbol{q}}(t)+\boldsymbol{B}_{i}^{\sigma}\boldsymbol{u}(t))+\boldsymbol{D}\overline{\boldsymbol{d}}(t) \tag{29}$$

$$\dot{\underline{\boldsymbol{q}}}(t)=\sum_{i=1}^{s}\mathrm{h}_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))(\underline{\boldsymbol{A}}_{i}^{\sigma}\underline{\boldsymbol{q}}(t)+\boldsymbol{B}_{i}^{\sigma}\boldsymbol{u}(t))+\boldsymbol{D}\underline{\boldsymbol{d}}(t) \tag{30}$$

and defining the upper and lower observer error vectors as follows

$$\overline{\boldsymbol{e}}(t)=\boldsymbol{q}(t)-\overline{\boldsymbol{q}}_e(t),\quad \underline{\boldsymbol{e}}(t)=\boldsymbol{q}(t)-\underline{\boldsymbol{q}}_e(t) \tag{31}$$

$$\overline{\boldsymbol{e}}_y(t)=\underline{\boldsymbol{C}}_{i}^{\sigma}\overline{\boldsymbol{e}}(t),\quad \underline{\boldsymbol{e}}_y(t)=\overline{\boldsymbol{C}}_{i}^{\sigma}\underline{\boldsymbol{e}}(t) \tag{32}$$

then, by subtracting (26) from (29), as well as by subtracting (27) from (30), the following dynamics can be obtained with $i\in\{1,\dots,s\}$, $\sigma\in\{1,\dots,n_s\}$.

$$\dot{\overline{\boldsymbol{e}}}(t)=\sum_{i=1}^{s}\mathrm{h}_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))\overline{\boldsymbol{A}}_{ei}^{\sigma}\overline{\boldsymbol{e}}(t)+\boldsymbol{D}\overline{\boldsymbol{d}}(t) \tag{33}$$

$$\dot{\underline{\boldsymbol{e}}}(t)=\sum_{i=1}^{s}\mathrm{h}_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\underline{\boldsymbol{A}}_{ei}^{\sigma}\underline{\boldsymbol{e}}(t)+\boldsymbol{D}\underline{\boldsymbol{d}}(t) \tag{34}$$

where the following notation is used

$$\overline{\boldsymbol{A}}_{ei}^{\sigma}=\overline{\boldsymbol{A}}_{i}^{\sigma}-\boldsymbol{J}_{i}^{\sigma}\underline{\boldsymbol{C}}^{\sigma},\quad \underline{\boldsymbol{A}}_{ei}^{\sigma}=\underline{\boldsymbol{A}}_{i}^{\sigma}-\boldsymbol{J}_{i}^{\sigma}\overline{\boldsymbol{C}}^{\sigma} \tag{35}$$

Remark 2: The used approach does not exclude the construction of M-T-S fuzzy switching interval observers for a class of M-T-S fuzzy switching system with defined interval boundaries on non-negative input matrix parameters $\overline{\boldsymbol{B}}_{i}^{\sigma}$, $\underline{\boldsymbol{B}}_{i}^{\sigma}$. In practice, however, this means in such a case to use a special method for the control synthesis with an insight into the interval input of the system. However, these non-negative input matrix parameters do not enter the interval observer synthesis conditions.

To simplify comparative interpretation of the proposed results, the following theorem for disturbance-free Metzler-Takagi-Sugeno fuzzy observer (not switched and with $\boldsymbol{d}(i) = \boldsymbol{0}$) is presented.

Theorem 2: [26] Assuming that $\overline{\boldsymbol{q}}(0)$, $\underline{\boldsymbol{q}}(0)$ are known and the initial state $\boldsymbol{q}(0)$ verifies that $\boldsymbol{0} \leq \underline{\boldsymbol{q}}(0) \leq \boldsymbol{q}(0) \leq \overline{\boldsymbol{q}}(0)$, then the matrices $\overline{\boldsymbol{A}}_{ei}$, $\underline{\boldsymbol{A}}_{ei} \in \mathbb{R}_{-+}^{n\times n}$ for all $i \in \{1,\ldots,s\}$ are strictly Metzler and Hurwitz if for given strictly Metzler matrices $\overline{\boldsymbol{A}}_i, \underline{\boldsymbol{A}}_i \in \mathbb{R}_{-+}^{n\times n}$ and non-negative matrices $\overline{\boldsymbol{C}}, \underline{\boldsymbol{C}} \in \mathbb{R}_{+}^{m\times n}$ there exist positive definite diagonal matrices $\boldsymbol{P}, \boldsymbol{V}_{ik} \in \mathbb{R}_{+}^{n\times n}$ such that for $h = 1,\ldots,n-1$ the followin inequalities are satisfied

$$\boldsymbol{P} \succ 0, \quad \boldsymbol{V}_{ik} \succ 0 \tag{36}$$

$$\boldsymbol{P}\overline{\boldsymbol{A}}_i + \overline{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{P} - \sum_{k=1}^{m}\boldsymbol{V}_{ik}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\underline{\boldsymbol{C}}_{dk} - \sum_{k=1}^{m}\underline{\boldsymbol{C}}_{dk}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{V}_{ik} \prec 0 \tag{37}$$

$$\boldsymbol{P}\underline{\boldsymbol{A}}_i + \underline{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{P} - \sum_{k=1}^{m}\boldsymbol{V}_{ik}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\overline{\boldsymbol{C}}_{dk} - \sum_{k=1}^{m}\overline{\boldsymbol{C}}_{dk}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{V}_{ik} \prec 0 \tag{38}$$

$$\boldsymbol{P}\overline{\boldsymbol{A}}_i(\nu,\nu) - \sum_{k=1}^{m}\boldsymbol{V}_{ik}\underline{\boldsymbol{C}}_{dk} \prec 0 \tag{39}$$

$$\boldsymbol{P}\underline{\boldsymbol{A}}_i(\nu,\nu) - \sum_{k=1}^{m}\boldsymbol{V}_{ik}\overline{\boldsymbol{C}}_{dk} \prec 0 \tag{40}$$

$$\boldsymbol{P}\boldsymbol{L}^h\overline{\boldsymbol{A}}_i(\nu+h,\nu)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=1}^{m}\boldsymbol{V}_{ik}\boldsymbol{L}^h\underline{\boldsymbol{C}}_{dk}\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{41}$$

$$\boldsymbol{P}\boldsymbol{L}^h\underline{\boldsymbol{A}}_i(\nu+h,p)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=1}^{m}\boldsymbol{V}_{ik}\boldsymbol{L}^h\overline{\boldsymbol{C}}_{dk}\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{42}$$

where

$$\boldsymbol{L} = \begin{bmatrix}\boldsymbol{0}^{\mathrm{T}} & 1\\ \boldsymbol{I}_{n-1} & \boldsymbol{0}\end{bmatrix},\ \overline{\boldsymbol{C}} = \begin{bmatrix}\overline{c}_{11}\ldots\overline{c}_{1n}\\ \vdots\\ \overline{c}_{m1}\ldots\overline{c}_{mn}\end{bmatrix},\ \underline{\boldsymbol{C}} = \begin{bmatrix}\underline{c}_{11}\cdots\underline{c}_{1n}\\ \vdots\\ \underline{c}_{m1}\cdots\underline{c}_{mn}\end{bmatrix} \tag{43}$$

$$\overline{\boldsymbol{C}}_{dk} = \mathrm{diag}\left[\overline{c}_{k1}\ldots\overline{c}_{kn}\right], \quad \underline{\boldsymbol{C}}_{dk} = \mathrm{diag}\left[\underline{c}_{k1}\cdots\underline{c}_{kn}\right] \tag{44}$$

$$\boldsymbol{l} = \begin{bmatrix}1\\ \vdots\\ 1\end{bmatrix},\ \overline{\boldsymbol{A}}_i = \begin{bmatrix}\overline{a}_{i11}\ldots\overline{a}_{i1n}\\ \vdots\\ \overline{a}_{in1}\ldots\overline{a}_{inn}\end{bmatrix},\ \underline{\boldsymbol{A}}_i = \begin{bmatrix}\underline{a}_{i11}\cdots\underline{a}_{i1n}\\ \vdots\\ \underline{a}_{in1}\cdots\underline{a}_{inn}\end{bmatrix} \tag{45}$$

$$\overline{\boldsymbol{A}}_i(\nu,\nu) = \mathrm{diag}\left[\overline{a}_{i11}\cdots\overline{a}_{inn}\right] \tag{46}$$

$$\underline{\boldsymbol{A}}_i(\nu,\nu) = \mathrm{diag}\left[\underline{a}_{i11}\cdots\underline{a}_{inn}\right] \tag{47}$$

$$\begin{aligned}&\overline{\boldsymbol{A}}_i(\nu+h,\nu)\\ &= \mathrm{diag}\left[\overline{a}_{i,1+h,1}\cdots\overline{a}_{i,n,n-h}\ \overline{a}_{i,1,n-h+1}\cdots\overline{a}_{ihn}\right]\end{aligned} \tag{48}$$

$$\begin{aligned}&\underline{\boldsymbol{A}}_i(\nu+h,\nu)\\ &= \mathrm{diag}\left[\underline{a}_{i,1+h,1}\cdots\underline{a}_{i,n,n-h}\ \underline{a}_{i,1,n-h+1}\cdots\underline{a}_{ihn}\right]\end{aligned} \tag{49}$$

When these conditions are successfully met then the rules to compute the set of strictly positive observer gains $\boldsymbol{J}_i \in \mathbb{R}_{+}^{n\times m}$ are

$$\boldsymbol{J}_{dik} = \boldsymbol{P}^{-1}\boldsymbol{V}_{ik},\ \boldsymbol{j}_{ik} = \boldsymbol{J}_{dik}\boldsymbol{l},\ \boldsymbol{J}_i = \left[\boldsymbol{j}_{i1}\cdots\boldsymbol{j}_{im}\right] \tag{50}$$

Remark 3: A strictly Metzler system matrix $\overline{\boldsymbol{A}} = \{\overline{a}_{ij}\}_{i,j=1}^{n}$, $\underline{\boldsymbol{A}} = \{\underline{a}_{lj}\}_{l,j=1}^{n}$, respectively, makes unnecessary considera-tion of n^2 constraints (for every interval parameter structure)

$$\overline{a}_{jj} < 0\ \forall\ j = 1,\ldots,n, \quad \overline{a}_{lj,l\neq j} > 0\ \forall\ l,j = 1,\ldots,n \tag{51}$$

$$\underline{a}_{jj} < 0\ \forall\ j = 1,\ldots,n, \quad \underline{a}_{lj,l\neq j} > 0\ \forall\ l,j = 1,\ldots,n \tag{52}$$

This just means in consequence to apply diagonal stabiliza-tion principle [27] in analysis. Such an assumption provides a new scheme to describe design conditions.

If a strictly Metzler $\boldsymbol{A} \in \mathbb{M}_{-+}^{n\times n}$ is represented with relation to the observer design task in the following rhombic form, where the diagonal local exactness are constructed by the column index defined multiple circular shifts of elements of the columns of $\boldsymbol{A}$ as follows [10]

$$\boldsymbol{A}_{\Theta} = \begin{bmatrix} a_{11} & & & & \\ a_{21} & a_{22} & & & \\ a_{31} & a_{32} & a_{33} & & \\ \vdots & \vdots & \vdots & \ddots & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn}\\ & a_{12} & a_{13} & \cdots & a_{1n}\\ & & a_{23} & \cdots & a_{2n}\\ & & & \ddots & \vdots\\ & & & & a_{n-1,n}\end{bmatrix} \tag{53}$$

then the diagonal matrix structures, related to $\boldsymbol{A}_{\Theta}$ with the index $h = 0,\ldots,n-1$

$$\boldsymbol{A}(\nu,\nu) = \mathrm{diag}\left[a_{11}\ a_{22}\ \cdots\ a_{nn}\right] \prec 0 \tag{54}$$

$$\boldsymbol{A}(\nu+h,\nu) = \mathrm{diag}\left[a_{1+h,1}\cdots a_{n,n-h}\ a_{1,n-h+1}\cdots a_{h,n}\right] \succ 0 \tag{55}$$

represent the set of Metzler parametric constraints

$$a_{jj} < 0\ \forall j = 1,\ldots,n, \quad a_{lj,l\neq j} > 0\ \forall\ l,j = 1,\ldots,n \tag{56}$$

Thus, generalising for (51), (52), the diagonal structures (46)-(49) can be defined.

Moreover, utilization of this principle leads to the Metzler matrix $\boldsymbol{A}$ parameterizations as [28]

$$\boldsymbol{A} = \sum_{h=0}^{n-1}\boldsymbol{A}(\nu+h,\nu)\boldsymbol{L}^{h\mathrm{T}} \tag{57}$$

where $\boldsymbol{L} \in \mathbb{R}^{n\times n}$ is the circulant permutation matrix (43).

Corollary 1: According to (57) the parameterizations of $\overline{\boldsymbol{A}}_{ei}^{\sigma}$, $\underline{\boldsymbol{A}}_{ei}^{\sigma}$ can be written as

$$\overline{\boldsymbol{A}}_{ei} = \sum_{h=0}^{n-1}\left(\overline{\boldsymbol{A}}_i^{\sigma}(\nu+h,\nu) - \sum_{k=0}^{m}\boldsymbol{J}_{ikh}^{\sigma}\underline{\boldsymbol{C}}_{dk}^{\sigma}\right)\boldsymbol{L}^{h\mathrm{T}} \tag{58}$$

$$\underline{\boldsymbol{A}}_{ei} = \sum_{h=0}^{n-1}\left(\underline{\boldsymbol{A}}_i^{\sigma}(\nu+h,\nu) - \sum_{k=0}^{m}\boldsymbol{J}_{ikh}^{\sigma}\overline{\boldsymbol{C}}_{dk}^{\sigma}\right)\boldsymbol{L}^{h\mathrm{T}} \tag{59}$$

where, with relation to $\boldsymbol{J}^{\sigma}$, $\underline{\boldsymbol{C}}^{\sigma}$, $\overline{\boldsymbol{C}}^{\sigma}$ the diagonal matrices $\boldsymbol{J}^{\sigma}_{ikh}, \underline{\boldsymbol{C}}^{\sigma}_{dk}, \overline{\boldsymbol{C}}^{\sigma}_{dk} \in \mathbb{R}^{n\times n}_{+}$ are defined as

$$\underline{\boldsymbol{C}}^{\sigma} = \begin{bmatrix} \underline{\boldsymbol{c}}^{\sigma\mathrm{T}}_1 \\ \vdots \\ \underline{\boldsymbol{c}}^{\sigma\mathrm{T}}_m \end{bmatrix}, \quad \underline{\boldsymbol{C}}^{\sigma}_{dk} = \mathrm{diag}\left[\underline{\boldsymbol{c}}^{\sigma\mathrm{T}}_k\right] \tag{60}$$

$$\overline{\boldsymbol{C}}^{\sigma} = \begin{bmatrix} \overline{\boldsymbol{c}}^{\sigma\mathrm{T}}_1 \\ \vdots \\ \overline{\boldsymbol{c}}^{\sigma\mathrm{T}}_m \end{bmatrix}, \quad \overline{\boldsymbol{C}}^{\sigma}_{dk} = \mathrm{diag}\left[\overline{\boldsymbol{c}}^{\sigma\mathrm{T}}_k\right] \tag{61}$$

$$\boldsymbol{J}^{\sigma}_{i} = \left[\boldsymbol{j}^{\sigma}_{i1} \cdots \boldsymbol{j}^{\sigma}_{im}\right], \ \boldsymbol{J}^{\sigma}_{ik} = \mathrm{diag}\left[\boldsymbol{j}^{\sigma}_{ik}\right], \ \boldsymbol{J}^{\sigma}_{ikh} = \boldsymbol{L}^{h\mathrm{T}}\boldsymbol{J}^{\sigma}_{ik}\boldsymbol{L}^{h} \tag{62}$$

and

$$\overline{\boldsymbol{A}}^{\sigma}_{i}(\nu,\nu) = \mathrm{diag}\left[\overline{a}^{\sigma}_{i11} \ \cdots \ \overline{a}^{\sigma}_{inn}\right] \tag{63}$$

$$\underline{\boldsymbol{A}}^{\sigma}_{i}(\nu,\nu) = \mathrm{diag}\left[\underline{a}^{\sigma}_{i11} \cdots \underline{a}^{\sigma}_{inn}\right] \tag{64}$$

$$\begin{aligned} &\overline{\boldsymbol{A}}^{\sigma}_{i}(\nu+h,\nu) \\ &= \mathrm{diag}\left[\overline{a}^{\sigma}_{i,1+h,1} \cdots \overline{a}^{\sigma}_{i,n,n-h} \ \overline{a}^{\sigma}_{i,1,n-h+1} \cdots \overline{a}^{\sigma}_{ihn}\right] \end{aligned} \tag{65}$$

$$\begin{aligned} &\underline{\boldsymbol{A}}^{\sigma}_{i}(\nu+h,\nu) \\ &= \mathrm{diag}\left[\underline{a}^{\sigma}_{i,1+h,1} \cdots \underline{a}^{\sigma}_{i,n,n-h} \ \underline{a}^{\sigma}_{i,1,n-h+1} \cdots \underline{a}^{\sigma}_{ihn}\right] \end{aligned} \tag{66}$$

Through these supporting formulations, the relationship between the M-T-S fuzzy switching interval dynamical system and the corresponding M-T-S fuzzy switching interval observer is derived in the following theorem.

Theorem 3: The matrices $\overline{\boldsymbol{A}}^{\sigma}_{ei}$, $\underline{\boldsymbol{A}}^{\sigma}_{ei} \in \mathbb{R}^{n\times n}_{-+}$ for all $i \in \{1,\ldots,s\}$, $\sigma \in \{1,\ldots,n_s\}$ are strictly Metzler and Hurwitz if for given strictly Metzler matrices, $\overline{\boldsymbol{A}}^{\sigma}_{i}, \underline{\boldsymbol{A}}^{\sigma}_{i} \in \mathbb{R}^{n\times n}_{-+}$, non-negative matrices $\overline{\boldsymbol{C}}, \underline{\boldsymbol{C}} \in \mathbb{R}^{m\times n}_{+}$ and for from these matrices derived diagonal matrix parameters (60)-(62) there exist positive definite diagonal matrices $\boldsymbol{P}, \boldsymbol{V}^{\sigma}_{ik} \in \mathbb{R}^{n\times n}_{+}$ and a positive scalar $\xi \in \mathbb{R}_{+}$ such that for $i = 1,\ldots,s$, $h = 1,\ldots,n-1$, $\sigma \in \{1,\ldots,n_s\}$

$$\boldsymbol{P} \succ 0, \quad \boldsymbol{V}^{\sigma}_{ik} \succ 0 \tag{67}$$

$$\begin{bmatrix} \overline{\boldsymbol{\Omega}}^{\sigma}_{i} & * & * \\ \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P} & -\xi\boldsymbol{I}_d & * \\ \underline{\boldsymbol{C}}^{\sigma} & \boldsymbol{0} & -\xi\boldsymbol{I}_m \end{bmatrix} \prec 0 \tag{68}$$

$$\begin{bmatrix} \underline{\boldsymbol{\Omega}}^{\sigma}_{i} & * & * \\ \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P} & -\xi\boldsymbol{I}_d & * \\ \overline{\boldsymbol{C}}^{\sigma} & \boldsymbol{0} & -\xi\boldsymbol{I}_m \end{bmatrix} \prec 0 \tag{69}$$

$$\boldsymbol{P}\overline{\boldsymbol{A}}^{\sigma}_{i}(\nu,\nu) - \sum_{k=1}^{m} \boldsymbol{V}^{\sigma}_{ik}\underline{\boldsymbol{C}}_{dk} \prec 0 \tag{70}$$

$$\boldsymbol{P}\underline{\boldsymbol{A}}^{\sigma}_{i}(\nu,\nu) - \sum_{k=1}^{m} \boldsymbol{V}^{\sigma}_{ik}\overline{\boldsymbol{C}}_{dk} \prec 0 \tag{71}$$

$$\boldsymbol{P}\boldsymbol{L}^{h}\overline{\boldsymbol{A}}^{\sigma}_{i}(\nu+h,\nu)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=1}^{m} \boldsymbol{V}^{\sigma}_{ik}\boldsymbol{L}^{h}\underline{\boldsymbol{C}}_{dk}\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{72}$$

$$\boldsymbol{P}\boldsymbol{L}^{h}\underline{\boldsymbol{A}}^{\sigma}_{i}(\nu+h,p)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=1}^{m} \boldsymbol{V}^{\sigma}_{ik}\boldsymbol{L}^{h}\overline{\boldsymbol{C}}_{dk}\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{73}$$

where

$$\overline{\boldsymbol{\Omega}}^{\sigma}_{i} = \boldsymbol{P}\overline{\boldsymbol{A}}^{\sigma}_{i} + \overline{\boldsymbol{A}}^{\sigma\mathrm{T}}_{i}\boldsymbol{P} - \sum_{k=1}^{m}\boldsymbol{V}^{\sigma}_{ik}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\underline{\boldsymbol{C}}_{dk} - \sum_{k=1}^{m}\underline{\boldsymbol{C}}_{dk}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{V}^{\sigma}_{ik} \tag{74}$$

$$\underline{\boldsymbol{\Omega}}^{\sigma}_{i} = \boldsymbol{P}\underline{\boldsymbol{A}}^{\sigma}_{i} + \underline{\boldsymbol{A}}^{\sigma\mathrm{T}}_{i}\boldsymbol{P} - \sum_{k=1}^{m}\boldsymbol{V}^{\sigma}_{ik}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\overline{\boldsymbol{C}}_{dk} - \sum_{k=1}^{m}\overline{\boldsymbol{C}}_{dk}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{V}^{\sigma}_{ik} \tag{75}$$

If the task is feasible, the rules to compute the set of strictly positive gains $\boldsymbol{J}^{\sigma}_{i} \in \mathbb{R}^{n\times m}_{+}$, $i \in \{1,\ldots,s\}$, $\sigma \in \{1,\ldots,n_s\}$, are

$$\boldsymbol{J}^{\sigma}_{dik} = \boldsymbol{P}^{-1}\boldsymbol{V}^{\sigma}_{ik}, \ \boldsymbol{j}^{\sigma}_{ik} = \boldsymbol{J}^{\sigma}_{dik}\boldsymbol{l}, \ \boldsymbol{J}^{\sigma}_{i} = \left[\boldsymbol{j}^{\sigma}_{i1} \cdots \boldsymbol{j}^{\sigma}_{im}\right] \tag{76}$$

Proof: According to the parametrization of $\overline{\boldsymbol{A}}^{\sigma}_{ei}$, $\underline{\boldsymbol{A}}^{\sigma}_{ei}$ (58), (59) it has to yield for all i, σ and $h=0$

$$\overline{\boldsymbol{A}}^{\sigma}_{i}(\nu,\nu) - \sum_{k=0}^{m} \boldsymbol{J}^{\sigma}_{ik}\underline{\boldsymbol{C}}^{\sigma}_{dk} \prec 0 \tag{77}$$

$$\overline{\boldsymbol{A}}^{\sigma}_{i}(\nu,\nu) - \sum_{k=0}^{m} \boldsymbol{J}^{\sigma}_{ik}\underline{\boldsymbol{C}}^{\sigma}_{dk} \prec 0 \tag{78}$$

and for all i, σ and $h = \{1,\ldots,n-1\}$

$$\overline{\boldsymbol{A}}^{\sigma}_{i}(\nu+h,\nu)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=0}^{m} \boldsymbol{J}^{\sigma}_{ikh}\underline{\boldsymbol{C}}^{\sigma}_{dk}\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{79}$$

$$\underline{\boldsymbol{A}}^{\sigma}_{i}(\nu+h,\nu)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=0}^{m} \boldsymbol{J}^{\sigma}_{ikh}\overline{\boldsymbol{C}}^{\sigma}_{dk}\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{80}$$

Multiplying by the positive definite diagonal matrix $\boldsymbol{P}$ the left side of (77), (78), respectively, it yields

$$\boldsymbol{P}\overline{\boldsymbol{A}}^{\sigma}_{i}(\nu,\nu) - \sum_{k=0}^{m} \boldsymbol{P}\boldsymbol{J}^{\sigma}_{ik}\underline{\boldsymbol{C}}^{\sigma}_{dk} \prec 0 \tag{81}$$

$$\boldsymbol{P}\overline{\boldsymbol{A}}^{\sigma}_{i}(\nu,\nu) - \sum_{k=0}^{m} \boldsymbol{P}\boldsymbol{J}^{\sigma}_{ik}\underline{\boldsymbol{C}}^{\sigma}_{dk} \prec 0 \tag{82}$$

and using the notation

$$\boldsymbol{V}^{\sigma}_{ik} = \boldsymbol{P}\boldsymbol{J}^{\sigma}_{ik} \tag{83}$$

then (81), (82) imply (70), (71).

Substituting (62) and multiplying by $\boldsymbol{P}\boldsymbol{L}^{h}$ the left side of (77), (78), respectively, now it yields

$$\boldsymbol{P}\boldsymbol{L}^{h}\overline{\boldsymbol{A}}^{\sigma}_{i}(\nu+h,\nu)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=0}^{m} \boldsymbol{P}\boldsymbol{J}^{\sigma}_{ik}\boldsymbol{L}^{h}\underline{\boldsymbol{C}}^{\sigma}_{dk}\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{84}$$

$$\boldsymbol{P}\boldsymbol{L}^{h}\underline{\boldsymbol{A}}^{\sigma}_{i}(\nu+h,\nu)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=0}^{m} \boldsymbol{P}\boldsymbol{J}^{\sigma}_{ik}\boldsymbol{L}^{h}\overline{\boldsymbol{C}}^{\sigma}_{dk}\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{85}$$

and with the notation (83) then (84), (85) imply (72), (73).

In the given sense then (70)-(73) force the Metzler parametric constraints in the design task.

Considering the common Lyapunov function (13) for the lower dynamic (31), (32) means

$$\begin{aligned} v(\underline{\boldsymbol{e}}(t)) &= \underline{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}\underline{\boldsymbol{e}}(t)+ \\ &+\xi^{-1}\int_0^t (\underline{\boldsymbol{e}}_y^{\mathrm{T}}(\tau)\underline{\boldsymbol{e}}_y(\tau)-\xi^2\,\underline{\boldsymbol{d}}^{\mathrm{T}}(\tau)\,\underline{\boldsymbol{d}}(\tau))\mathrm{d}\tau \\ &>0 \end{aligned} \tag{86}$$

and in the same way of the proof of Theorem 1 it follows that

$$\begin{aligned} \dot{v}(\underline{\boldsymbol{e}}(t)) &= \dot{\underline{\boldsymbol{e}}}^{\mathrm{T}}(t)\boldsymbol{P}\underline{\boldsymbol{e}}(t)+\underline{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}\underline{\dot{\boldsymbol{e}}(t)}+ \\ &+\xi^{-1}\underline{\boldsymbol{e}}_y^{\mathrm{T}}(t)\underline{\boldsymbol{e}}_y(t)-\xi\,\underline{\boldsymbol{d}}^{\mathrm{T}}(t)\underline{\boldsymbol{d}}(t) \\ &<0 \end{aligned} \tag{87}$$

$$\begin{aligned} \dot{v}(\underline{\boldsymbol{e}}(t)) &= \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\underline{\boldsymbol{e}}^{\mathrm{T}}(t)(\underline{\boldsymbol{A}}_{ei}^{\sigma\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\underline{\boldsymbol{A}}_{ei}^{\sigma})\underline{\boldsymbol{e}}(t)+ \\ &+\underline{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{D}\underline{\boldsymbol{d}}(t)+\underline{\boldsymbol{d}}^{\mathrm{T}}(t)\boldsymbol{D}^{\mathrm{T}}\boldsymbol{P}\underline{\boldsymbol{e}}(t))+ \\ &+\xi^{-1}\underline{\boldsymbol{e}}^{\mathrm{T}}(t)\overline{\boldsymbol{C}}^{\sigma\mathrm{T}}\overline{\boldsymbol{C}}^{\sigma}\underline{\boldsymbol{e}}(t)-\xi\underline{\boldsymbol{d}}^{\mathrm{T}}(t)\underline{\boldsymbol{d}}(t) \\ &<0 \end{aligned} \tag{88}$$

Analogously to (16) by defining

$$\underline{\boldsymbol{e}}_d^{\mathrm{T}}(t) = \begin{bmatrix}\underline{\boldsymbol{e}}^{\mathrm{T}}(t) & \underline{\boldsymbol{d}}^{\mathrm{T}}(t)\end{bmatrix} \tag{89}$$

it can be written that

$$\dot{v}(\underline{\boldsymbol{e}}_d(t)) = \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\underline{\boldsymbol{e}}_d^{\mathrm{T}}(t)\,\underline{\boldsymbol{\Xi}}_i^{\sigma}\,\underline{\boldsymbol{e}}_d(t)<0 \tag{90}$$

where, by the used labeling,

$$\underline{\boldsymbol{\Xi}}_i^{\sigma} = \begin{bmatrix} \boldsymbol{P}\underline{\boldsymbol{A}}_{ei}^{\sigma}+\underline{\boldsymbol{A}}_{ei}^{\sigma\mathrm{T}}\boldsymbol{P}+\xi^{-1}\overline{\boldsymbol{C}}^{\sigma\mathrm{T}}\overline{\boldsymbol{C}}^{\sigma} & * \\ \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P} & -\xi\boldsymbol{I}_d \end{bmatrix} \prec 0 \tag{91}$$

and, consequently, the structure (69) results using Schur complement property to (91)

$$\begin{bmatrix} \underline{\boldsymbol{\Omega}}_i^{\sigma} & * & * \\ \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P} & -\xi\boldsymbol{I}_d & * \\ \overline{\boldsymbol{C}}^{\sigma} & \boldsymbol{0} & -\xi\boldsymbol{I}_m \end{bmatrix} \prec 0 \tag{92}$$

where

$$\underline{\boldsymbol{\Omega}}_i^{\sigma} = \boldsymbol{P}\underline{\boldsymbol{A}}_{ei}^{\sigma}+\underline{\boldsymbol{A}}_{ei}^{\sigma\mathrm{T}}\boldsymbol{P} \tag{93}$$

Substituting (35) into (93) and using the properties (60), (50) then

$$\begin{aligned} &\underline{\boldsymbol{\Omega}}_i^{\sigma} \\ =&\boldsymbol{P}(\underline{\boldsymbol{A}}_i^{\sigma}-\boldsymbol{J}_i^{\sigma}\overline{\boldsymbol{C}}^{\sigma})+(\underline{\boldsymbol{A}}_i^{\sigma}-\boldsymbol{J}_i^{\sigma}\overline{\boldsymbol{C}}^{\sigma})^{\mathrm{T}}\boldsymbol{P} \\ =&\boldsymbol{P}\Big(\underline{\boldsymbol{A}}_i^{\sigma}-\sum_{k=1}^{m}\boldsymbol{j}_{ik}^{\sigma}\overline{\boldsymbol{c}}_k^{\sigma\mathrm{T}}\Big)+\Big(\underline{\boldsymbol{A}}_i^{\sigma}-\sum_{k=1}^{m}\boldsymbol{j}_{ik}^{\sigma}\overline{\boldsymbol{c}}_k^{\sigma\mathrm{T}}\Big)^{\mathrm{T}}\boldsymbol{P} \\ =&\boldsymbol{P}\Big(\underline{\boldsymbol{A}}_i^{\sigma}-\sum_{k=1}^{m}\boldsymbol{J}_{ik}^{\sigma}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\overline{\boldsymbol{C}}_{dk}^{\sigma}\Big)+\Big(\underline{\boldsymbol{A}}_i^{\sigma}-\sum_{k=1}^{m}\boldsymbol{J}_{ik}^{\sigma}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\overline{\boldsymbol{C}}_{dk}^{\sigma}\Big)^{\mathrm{T}}\boldsymbol{P} \end{aligned} \tag{94}$$

Thus, applying (83) to (94) then (75) implies.

Considering the common Lyapunov function (13) for the upper dynamics (31), (32), the condition (68) and (74) can be proven similarly. This concludes the proof. ■

IV. Illustrative Example

The system is represented by the M-T-S fuzzy switching model (1), (2), where [23]

$$\underline{\boldsymbol{A}}_1^1 = \begin{bmatrix} -0.272 & 1.940 & 1.450 \\ 0.058 & -3.960 & 0.050 \\ 0.100 & 0.050 & -2.910 \end{bmatrix}, \quad \underline{\boldsymbol{C}}^1 = \underline{\boldsymbol{C}}^2 = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 1.2 & 0 \end{bmatrix}$$

$$\underline{\boldsymbol{A}}_2^1 = \begin{bmatrix} -0.272 & 1.940 & 1.450 \\ 0.058 & -3.960 & 0.100 \\ 0.100 & 0.050 & -2.910 \end{bmatrix}, \quad \boldsymbol{L} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\underline{\boldsymbol{A}}_1^2 = \begin{bmatrix} -0.272 & 1.940 & 1.450 \\ 0.058 & -3.960 & 0.050 \\ 0.100 & 0.080 & -2.910 \end{bmatrix}$$

$$\underline{\boldsymbol{A}}_2^2 = \begin{bmatrix} -0.272 & 1.940 & 1.450 \\ 0.058 & -3.960 & 0.100 \\ 0.100 & 0.080 & -2.910 \end{bmatrix}$$

$$\overline{\boldsymbol{A}}_1^1 = \begin{bmatrix} -0.258 & 2.060 & 1.550 \\ 0.142 & -3.640 & 0.060 \\ 0.200 & 0.070 & -2.550 \end{bmatrix}, \quad \overline{\boldsymbol{C}}^1 = \overline{\boldsymbol{C}}^2 = \begin{bmatrix} 1.1 & 0 & 0 \\ 0 & 1.5 & 0 \end{bmatrix}$$

$$\overline{\boldsymbol{A}}_2^1 = \begin{bmatrix} -0.258 & 2.060 & 1.550 \\ 0.142 & -3.640 & 0.100 \\ 0.200 & 0.060 & -2.550 \end{bmatrix}, \quad \boldsymbol{l} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\overline{\boldsymbol{A}}_1^2 = \begin{bmatrix} -0.258 & 2.060 & 1.550 \\ 0.142 & -3.640 & 0.070 \\ 0.200 & 0.080 & -2.550 \end{bmatrix}$$

$$\overline{\boldsymbol{A}}_2^2 = \begin{bmatrix} -0.258 & 2.060 & 1.550 \\ 0.142 & -3.640 & 0.100 \\ 0.200 & 0.080 & -2.550 \end{bmatrix}$$

$$\boldsymbol{D}^{\mathrm{T}} = \begin{bmatrix} 0.04 & 0.08 & 0.05 \end{bmatrix}, \ \overline{d} = 0.7, \ \underline{d} = -\overline{d}$$

It is not hard to attest that $\overline{\boldsymbol{A}}_i^{\sigma}$, $\underline{\boldsymbol{A}}_i^{\sigma}$ are strictly Metzler and Hurwitz for all i, σ, $\underline{\boldsymbol{A}}_i^{\sigma} \leq \overline{\boldsymbol{A}}_i^{\sigma}$, $\underline{\boldsymbol{C}} \leq \overline{\boldsymbol{C}}$, and $\underline{\boldsymbol{C}}$, $\overline{\boldsymbol{C}}$ are nonnegative matrices.

The diagonal stabilization principle call for associated diagonal representations of $\underline{\boldsymbol{C}}$, $\overline{\boldsymbol{C}}$

$$\underline{\boldsymbol{C}}_{d1} = \operatorname{diag}\begin{bmatrix} 0.9 & 0 & 0 \end{bmatrix}, \quad \underline{\boldsymbol{C}}_{d2} = \operatorname{diag}\begin{bmatrix} 0 & 0.2 & 0 \end{bmatrix}$$
$$\overline{\boldsymbol{C}}_{d1} = \operatorname{diag}\begin{bmatrix} 1.1 & 0 & 0 \end{bmatrix}, \quad \overline{\boldsymbol{C}}_{d2} = \operatorname{diag}\begin{bmatrix} 0 & 0.5 & 0 \end{bmatrix}$$

and for circulant diagonal representations of the matrices $\underline{\boldsymbol{A}}_i^{\sigma}$ where for illustration

$$\underline{\boldsymbol{A}}_1^1(\nu,\nu) = \operatorname{diag}\begin{bmatrix} -0.272 & -3.960 & -2.910 \end{bmatrix}$$
$$\underline{\boldsymbol{A}}_1^1(\nu+1,\nu) = \operatorname{diag}\begin{bmatrix} 0.058 & 0.050 & 1.450 \end{bmatrix}$$
$$\underline{\boldsymbol{A}}_1^1(\nu+2,\nu) = \operatorname{diag}\begin{bmatrix} 0.100 & 1.940 & 0.050 \end{bmatrix}.$$

Using the toolbox SeDuMi [29] means construction of $N = 42$ matrix inequalities, from which $N_{mv} = 10$ prescribe the positivity of the matrix variables, $N_{st} = 8$ declare the Lyapunov stability condition and $N_{pb} = 24$ define the parametric boundaries for final Metzler matrix in the observer's structure.

The feasible solution of (67)-(75) produces LMI variables as follows

$$\boldsymbol{P} = \text{diag}\left[4.1339\ 2.6156\ 4.4060\right], \quad \xi = 6.9236$$

$$\boldsymbol{V}_{11}^{1} = \text{diag}\left[4.8067\ 0.0509\ 0.1224\right]$$

$$\boldsymbol{V}_{12}^{1} = \text{diag}\left[3.0868\ 1.3516\ 0.0568\right]$$

$$\boldsymbol{V}_{21}^{1} = \text{diag}\left[4.8080\ 0.0509\ 0.1225\right]$$

$$\boldsymbol{V}_{22}^{1} = \text{diag}\left[3.0879\ 1.3514\ 0.0541\right]$$

$$\boldsymbol{V}_{11}^{2} = \text{diag}\left[4.8076\ 0.0509\ 0.1225\right]$$

$$\boldsymbol{V}_{12}^{2} = \text{diag}\left[3.0873\ 1.3515\ 0.0806\right]$$

$$\boldsymbol{V}_{21}^{2} = \text{diag}\left[4.8091\ 0.0510\ 0.1226\right]$$

$$\boldsymbol{V}_{22}^{2} = \text{diag}\left[3.0885\ 1.3515\ 0.0806\right]$$

whilst (76) implies the positive observer gain matrices

$$\boldsymbol{J}_1^1 = \begin{bmatrix} 1.1627 & 0.7467 \\ 0.0194 & 0.5167 \\ 0.0278 & 0.0129 \end{bmatrix}, \ \boldsymbol{J}_2^1 = \begin{bmatrix} 1.1630 & 0.7467 \\ 0.0195 & 0.5166 \\ 0.0278 & 0.0129 \end{bmatrix}$$

$$\boldsymbol{J}_1^2 = \begin{bmatrix} 1.1630 & 0.7468 \\ 0.0196 & 0.5167 \\ 0.0278 & 0.0183 \end{bmatrix}, \ \boldsymbol{J}_2^1 = \begin{bmatrix} 1.1633 & 0.7471 \\ 0.0195 & 0.5167 \\ 0.0278 & 0.0183 \end{bmatrix}$$

guaranteing all Metzler and Hurwitz stable matrices of the switched interval observer.

For illustration

$$\underline{\boldsymbol{A}}_{e1}^{1} = \begin{bmatrix} -1.5510 & 0.8200 & 1.4500 \\ 0.0366 & -4.7351 & 0.0500 \\ 0.0694 & 0.0307 & -2.9100 \end{bmatrix}, \ \rho(\underline{\boldsymbol{A}}_{e1}^{1}) = \begin{Bmatrix} -1.4709 \\ -2.9805 \\ -4.7447 \end{Bmatrix}$$

$$\overline{\boldsymbol{A}}_{e1} = \begin{bmatrix} -1.3045 & 1.1640 & 1.5500 \\ 0.1245 & -4.2601 & 0.0600 \\ 0.1750 & 0.0545 & -2.5500 \end{bmatrix}, \ \rho(\overline{\boldsymbol{A}}_{e1}^{1}) = \begin{Bmatrix} -1.0707 \\ -2.7355 \\ -4.3083 \end{Bmatrix}$$

where. evidently, $\underline{\boldsymbol{A}}_{e1}^{1} < \overline{\boldsymbol{A}}_{e1}^{1}$.

Having in mind (22) it is not hard to verify that the condition $\underline{\boldsymbol{A}}_{ei}^{\sigma} \leq \overline{\boldsymbol{A}}_{ei}^{\sigma}$ is satisfied for all $i = 1, 2$, $\sigma = 1, 2$.

Note, the small complexity of the LMI algorithmic problems and LMIs feasibility can be checked also when programming the task using the LMI toolbox of MATLAB©.

The method can be easily adapted exploiting structured matrix variables for design of M-T-S fuzzy switched interval observers for purely Metzler system matrices, where the off-diagonal elements of a Metzler matrix are non-negative, by using the method presented in [30]. In such a case the solution results in existence of nonnegative $\boldsymbol{J}_i^{\sigma}$, guaranteing purely Metzler and Hurwitz switched observer's matrices $\underline{\boldsymbol{A}}_{ei}^{\sigma}, \overline{\boldsymbol{A}}_{ei}^{\sigma}$.

As can be seen from the example, the proposed procedure transforms the problem of M-T-S fuzzy switched interval observers design to convenient LMI forms.

V. Concluding Remarks

By utilising information of both the upper bound and the lower bound of premise variables and their actual real-time measurement, it is formulated a finite number of LMIs to prescribe nonnegative interval observer gains in construction of the system Metzler and Hurwitz matrices in the design task for interval switched M-T-S fuzzy observers. The novelty lies in a common LMI-based encompassing of interval bounds, Metzler matrix parameters and stability conditions in the problem formulation. The implementation exploits by this way defined procedures to manipulate the interval switched M-T-S fuzzy observer stability and the Metzler structural bounds, as well as to guarantee convergence to equilibria of the estimation errors. The presented example indicates that the defined LMI forms are applicable in synthesis of the interval observers for uncertain M-T-S fuzzy systems.

Generalizing interval state estimation of switched M-T-S fuzzy observers, further research can be potentially focused on positive control of agent systems and stochastic nonlinear switched systems. The same methods of solution can be motivation to the algorithmic support when designing the interval observers for fractional T-S fuzzy switched systems.

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