The Over-Damped String Stability Condition for a Platooning System

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***Abstract*—The over-damped string stability criterion is a very strong stability condition that not only addresses the stability in a stricter sense but also adequately captures the safety performance of a platoon. However, the mathematical representation of this criterion is incomplete in the literature. Here, this representation is completely described. Moreover, this article presents the mathematical test method to evaluate this stability condition for linear or linearized systems from the transfer function. The classical sting stability condition does not address the transient undesired convergent dynamics of a platoon, such as over-shooting, under-shooting or damped oscillating dynamics. This paper demonstrates that the over-damped string stability characteristic significantly attenuates these undesired convergent dynamics in the upstream direction. Thus, the advantage of this condition over the classical criterion for linear system is clarified theoretically and by simulation. Later, the numerical method to analyze the over-damped string stability criterion for nonlinear systems is discussed. Additionally, numerical simulations of an over-damped string stable adaptive cruise control (ACC) vehicle model are compared with that of some experimental test results on platoons of commercially implemented ACC equipped vehicles.**

***Index Terms*—over-damped string stability, classical string stability, platooning control, adaptive cruise control**

I. Introduction

The stability of automated platooning systems plays a significant role in enhancing the traffic safety and performance [1]. The adaptive cruise control system (ACC) or the cooperative adaptive cruise control (CACC) systems can be effectively used to form a stable platoon. Definitely, when considering the performance of an individual vehicle, the local or individual vehicle stability is a primary requirement for such ACC/CACC systems [1], [2]. However, when the stability of a platooning system is considered, the string stability criterion is a necessary condition, which addresses the performance of the platoon collectively. Classically, a platoon is string stable when an external perturbation in the string does not amplify in the upstream direction [3]. In the traffic engineering literature, an upstream direction is referred to the direction opposite to the direction of motion of the vehicles in the considered platoon. This classical string stability condition is mostly discussed in the literature, see e.g., [4]–[6]. One of the major drawbacks of this condition is that it does not address the transient convergence dynamics of the string, when the stable system converges to a steady-state from the external perturbation, like over-shoot, under-shoot, or damped oscillations [2]. These undesired transient dynamics negatively affect the safety and performance of the platooning system, even when the platoon fulfills the classical string stability criterion.

The over-damped string stability norm not only includes the local and the classical string stability conditions but also ensures that, if the condition is satisfied, the platoon is free from the undesired transient convergence effects, i.e., over-shoot, under-shoot, and damped oscillations [2]. This criterion is fulfilled if the platooning system is externally positive, i.e., when the impulse response of the input-output transfer function of the system is never less than zero [2], [7]. An externally positive platooning system is also free from collision dynamics [8]–[10], thus it guarantees safety. Furthermore, the over-damped string stability criterion includes all the L_p, $p \in [1, \infty]$ norms [11], where L_∞ is the most restrictive norm [12]. Therefore, the over-damped string stability criterion is a complete and a very strong stability condition that adequately addresses the safety of an automated platooning system.

The over-damped string stability condition has been recently introduced, see [2], [7], [11], [13]. In [2], it is demonstrated that this condition includes both the over-damped local and classical string stability conditions. In other words, if the over-damped string stability is satisfied, the system holds the over-damped local and classical string stability criteria. In [7], a general design methodology is presented to derive an ACC system that fulfills this over-damped string stability norm,

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suitable for any linear or convex velocity dependent spacing function. The presented strategy compensates the negative effect of the lower-level time-lag on the stability. In [11], the adaptive time gap (ATG) car-following model [14], i.e., a special ACC algorithm, is extended in order to fulfill this strong string stability condition, when the lower-level longitudinal vehicle dynamics is considered. In these articles, the importance of this over-damped string stability norm is only briefly discussed, because the central focus of these articles is not on the nature and dynamics of this condition, but to use this condition as a primary design criterion. An in-depth analysis of this condition with respect to its dynamics and by using empirical data is still missing in the literature. In this paper we aim to fill this gap. Moreover, in the existing articles excluding [13], the mathematical representation of this criterion has a certain loophole. Therefore, the mathematical representation is extended in this article to overcome the inadequacy. This paper is an extension of our conference contribution [15].

This paper is organized as follows: The necessary background theories related to this contribution are discussed in Section II. Section III presents the analytical test method for the over-damped string stability condition from the transfer function (TF) of the linear or linearized system. In Section IV, the physical nature of both the classical and over-damped string stability norms are presented. Section V illustrates that a classical string stable platooning system could result in traffic oscillations and congestion when the over-damped stability condition is not fulfilled. This section also presents simulation results of a linear homogeneous platoon to illustrate the importance of the over-damped string stability condition. Section VI presents the numerical validation test for investigating the over-damped string stable characteristics for nonlinear systems. In Section VII the performance of an over-damped string stable ACC model is presented using real driving data of the leading vehicle from the experimental test data by Gunter et al. [16] and OpenACC database [17]. From these tests, the suitable experimental driving tests to analyze the collective over-damped dynamic feature is clarified. Also, in this section a comparison of the performance of the string stable ACC model with that of some commercially available ACC systems is shown. Finally, the paper is concluded in Section VIII.

II. Platooning Systems

A. Kinematics and Dynamics of the Vehicles

A platooning system can be formed by a string of autonomous ACC/CACC equipped vehicles. The use of the ACC systems for a platoon is still a plausible solution, as the systems are independent of the inter-vehicle communication signals and their topologies. Fig. 1 shows the nomenclature of two consecutive vehicles in a platoon and their respective kinematic variables and parameters associated with the longitudinal motion. Here, the index 'j' represents the considered ACC equipped vehicle and '$j+1$' represents the immediately preceding vehicle. The variables x and v represent the instantaneous position and velocity of the indexed vehicle, respectively. The constant parameters l and S represent the corresponding longitudinal length and the required stand-still spacing between two consecutive vehicles in the platoon, respectively. The headway spacing, excluding the stand-still distance, is expressed by

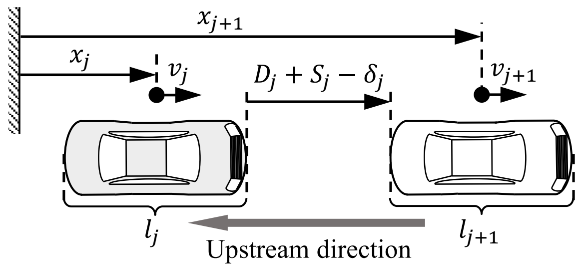

Fig. 1. Two consecutive vehicles in a platoon.

$$\Delta x_j = x_{j+1} - x_j - \left\{ S_j + \frac{l_j + l_{j+1}}{2} \right\} . \tag{1}$$

In Fig. 1, D_j represents the desired spacing between the vehicles. This value is generally dependent on the velocity of the vehicle, and the function $D_j(v_j)$ can be linear as well as nonlinear, depending on the choice of the spacing policy [5]. The spacing error between the j^{th} and $(j+1)^{\text{th}}$ vehicle is

$$\delta_j = D_j - \Delta x_j \, . \tag{2}$$

The ACC algorithm computes a desired acceleration u from the feedback signals, illustrated by

$$u_j = F\left(\Delta x_j, v_j, v_{j+1}, ...\right) . \tag{3}$$

This desired acceleration u is converted to the actual acceleration a by the power-train and braking module which constitute the longitudinal lower-level vehicle dynamics. This dynamics is typically represented by a first-order system, given by

$$\tau \, \dot{a}_j + a_j = u_j \, , \tag{4}$$

where τ is the time-lag of this lower control-loop, see [1], [7], [18].

B. The ACC System

In the following, for simplicity and without any loss of generality, the constant time-gap (CTG) spacing function is considered [3], [5]. The dynamic difference between the over-damped and only classically string stable characteristics is clearly noticeable even for nonlinear models [7], [11]. However, the CTG spacing function is simpler to analyze and, thus, the generality in terms of dynamic stability characteristics would not be lost. In this contribution, a time-lag compensating ACC function presented in [3], [4], [7], [11] is considered. The method used in [7], [11] provides an explicit boundary condition for the over-damped string stability norm, such that a clear analysis of the platooning systems can be demonstrated. When this strategy is applied, the desired spacing for a CTG spacing function reads

$$D_j = T \, v_j + T_a^2 \, a_j \, , \tag{5}$$

where T is the constant time-gap and T_a is a constant corresponding to an anticipation time [7], [11]. This anticipation time T_a is used to compensate the time-lag τ of the lower control-loop, see [11]. The desired acceleration computed by the ACC using this strategy is

$$u_j = \left(1 - \frac{\tau T}{T_a^2}\right) a_j + \frac{\tau}{T_a^2} \left(\Delta v_j - \lambda \delta_j\right) , \tag{6}$$

where Δv_j is the relative velocity equal to $(v_{j+1} - v_j)$ and λ is a positive constant, for details see [3], [4], [7].

C. String Stability

The classical string stability condition is expressed by the following input-output relationship:

$$\|H(j\omega)\|_\infty = \sup_\omega |H(j\omega)| \;\leq\; 1 , \tag{7}$$

where $H(s)$ is the TF of the system. For a linear homogeneous platooning system

$$H(s) = \frac{V_j(s)}{V_{j+1}(s)} , \tag{8}$$

where $V(s)$ represents the respective velocity in the Laplace domain [2]–[4], [7], [11]. The over-damped string stability condition is

$$h(t) \;\geq\; 0 \quad \& \quad \lim_{t\to\infty} h(t) = 0 \qquad \forall\, t \geq 0 , \tag{9}$$

where $h(t)$ is the impulse response of the TF $H(s)$ [2], [7], [11], [13]. Note that in the existing literature the additional part with the limit is missing, see [2], [7], [11], [15], except in [13]. Without this part the over-damped string stability condition would not be accurate, as an unstable system could also have a non-negative impulse response, e.g., the TF

$$H(s) = \frac{p_1}{s - p_1} \qquad p_1 > 0 . \tag{10}$$

With this extension, the condition (9) rules out the unstable TF and the expression precisely states that the system settles to the steady-state, when left undisturbed during the transient-state.

III. Mathematical Test Method for the Over-damped String Stability Condition for Linear or Linearized Systems

The analysis involving TFs only applies to the linear time-invariant (LTI) systems. Therefore, to perform such analysis, the system should either be linear or linearized across a chosen equilibrium point of linearization (CEPL) [7]. For the analyses of the condition (9), it is not at all necessary to perform the time-domain conversion of the TF (8) using inverse Laplace transform. At the same time, it is not always easy to conclude on the compliance of (9) from the time-domain function $h(t)$. Indeed the properties and the locations of the poles and zeros (if existing) in the complex s-plane can precisely describe the fulfillment of the condition (9).

The impulse response $h(t)$ of a TF $H(s)$ would be non-negative for all $t \geq 0$ and settle to zero during the steady-state, firstly, when all the poles and zeros (if existing) are real and negative, i.e., [7], [13]

$$\text{poles}\,\{H(s)\} \;\; \& \;\text{zeros}\,\{H(s)\} \in \mathbb{R} < 0 . \tag{11}$$

Secondly, the total number of zeros n should not be higher than the total number of poles m, i.e., [13]

$$n \leq m . \tag{12}$$

Finally, if zeros exist, these should be smaller than or equal to the corresponding poles, i.e.,

$$z_k \leq p_i , \tag{13}$$

where p_i and z_k are ordered poles and zeros, respectively, such that

$$p_1 \geq p_2 \geq ... \geq p_i \geq ... \geq p_m , \quad i = \{1, 2, ..., m\} , \tag{14a}$$
$$z_1 \geq z_2 \geq ... \geq z_k \geq ... \geq z_n , \quad k = \{1, 2, ..., n\} , \tag{14b}$$
$$s.t. \quad m \geq n ,$$

see [13], [19]. The following subsections provide some common examples for TFs of different orders.

A. First-order System

For a TF of first-order without zero,

$$H(s) = \frac{|p_1|}{s - p_1} , \tag{15}$$

the conditions (12) and (13) do not apply, and the system strictly fulfills (9) when p_1 is negative, as described by (11) [13]. On the other hand, for a TF

$$H(s) = \left|\frac{p_1}{z_1}\right| \, \frac{s - z_1}{s - p_1} , \tag{16}$$

from (11) and (13), the over-damped string stability is fulfilled if z_1 and p_1 are both negative, such that $z_1 \leq p_1$.

B. Second-order System

The standard second-order TF used in the control theory literature is given by

$$H\,(s) = \frac{\omega_0^2}{s^2 + 2\,\xi\,\omega_0\,s + \omega_0^2} , \tag{17}$$

where ω_0 is the undamped natural frequency of the system and ξ is the damping ratio [20]. From (11), the poles are real and negative, firstly, when ω_0 and ξ are both positive. Secondly, when the roots of the quadratic characteristic equation are real, this is possible when

$$\xi \geq 1 . \tag{18}$$

Hence, the condition (18) guarantees the over-damped string stability for a second-order system represented by (17) when ω_0 is positive.

C. Third-order System

The higher the order, the higher is the complexity of analyses. Consider the third-order TF

$$H(s) = \left|\frac{c}{d}\right| \frac{s+d}{s^3 + a\,s^2 + b\,s + c}\,. \tag{19}$$

For certain ACC functions, the closed-loop TF could result in this structure if the time-lag τ is not compensated, see [7]. First of all, to partially fulfill (11), all the parameters a, b, c, and d should be positive. This ensures that the three poles (yet undetermined) and the single zero (i.e., $-d$) are on the left half of the complex s-plane. Then the condition (11) is fully satisfied when the roots of the characteristic equation are real. The roots of the third-order polynomial would be real only when

$$a^2b^2 - 4\,b^3 - 4\,a^3c + 18\,a\,b\,c - 27\,c^2 \;\geq\; 0\,, \tag{20}$$

see [21]. Additionally, from (13), the closed-loop system (19) would be over-damped string stable if

$$d \;\geq\; -\max\left[\text{roots}\left\{s^3 + a\,s^2 + b\,s + c = 0\right\}\right]\,, \tag{21}$$

see [7], [11]. In a nutshell, the third-order system (19) would fulfill over-damped string stability condition (9), when all the parameters are positive such that both (20) and (21) hold true.

IV. Physical Nature of the String Stability Conditions

Note that the closed-loop TF for the ACC system (6) is of second-order, i.e.,

$$H\,(s) = \frac{1}{T_a^2\,s^2 + T\,s + 1}\,, \tag{22}$$

see [3], [7], [11]. The TF (22) is not dependent on the time-lag parameter τ. Therefore, the time-lag is compensated. The classical string stability condition for the TF (22) is

$$0 \;<\; T_a \;\leq\; \frac{T}{\sqrt{2}}\,, \tag{23}$$

and the over-damped string stability condition is

$$0 \;<\; T_a \;\leq\; \frac{T}{2}\,, \tag{24}$$

for details refer to [7], [11]. Therefore, the over-damped string stability condition also satisfies the classical string stability condition. When the parameter T_a is within

$$\frac{T}{2} \;<\; T_a \;\leq\; \frac{T}{\sqrt{2}}\,, \tag{25}$$

the ACC system (6) only satisfies the classical string stability and not the over-damped string stability norm. This region is briefly discussed in [7]. It is shown that the under-shooting transient effect can amplify upstream, even if the system is classical string stable. In this paper, we aim to present the analysis of this region in a greater depth.

Comparing (22) and (17), we get

$$\omega_0 = \frac{1}{T_a}\,, \tag{26}$$

and

$$\xi = 0.5\,\frac{T}{T_a}\,. \tag{27}$$

Therefore, the additional parameter T_a, added to the CTG spacing function (5), is actually the inverse of the natural frequency of the closed loop ACC systems. Also, the ratio of the time-gap T to the anticipation time T_a determines the damping nature of the system.

The classical string stability given by (7) means that the amplitude of the frequency response does not have maxima for all $\omega > 0$. The frequency response of the second order system (17) would not have a maximum, when the system does not have a resonance frequency. The resonance frequency of the TF (17) is

$$\omega_r = \omega_0\,\sqrt{1 - 2\,\xi^2}\,, \tag{28}$$

see [20]. The system would have no resonance frequency when ω_r is zero or a pure imaginary number, this is possible when

$$\xi \geq \frac{1}{\sqrt{2}}\,. \tag{29}$$

Substituting (27) to (29) results in the classical string stability condition given by (23). This condition only addresses the amplitude of the system in frequency domain and hence does not provide any information related to the convergence behavior of the system in the transient state.

It is already discussed in Section III-B that the second-order system (17) is over-damped when the damping ratio ξ is equal to or greater than 1, i.e., (18). Now, substituting (27) to (18) results in the over-damped string stability condition given by (24). The system (17) is not over-damped but only satisfies the classical string stability norm, when the damping ratio is within the range:

$$\frac{1}{\sqrt{2}} \;\leq\; \xi \;<\; 1\,, \tag{30}$$

which results in (25).

V. Illustration of the String Stability Characteristics by Simulation Test

In a simulation test, a leading vehicle is followed by 43 ACC equipped vehicles with homogeneous parameters. In the test, the leading vehicle in the platoon suddenly decelerates from a steady slow velocity of 8 m/s to a walking speed of 1 m/s, with a deceleration of -5 m/s^2.

Fig. 2 shows the simulation result of the system that only fulfills the classical string stability condition and not the over-damped stability norm, i.e., the parameter T_a is chosen according to (25). The value of T_a and the other parameters that are used to describe this platooning system are

$$\begin{bmatrix}\tau\\ \lambda\\ T\\ T_a\end{bmatrix} = \begin{bmatrix}0.80\,s\\ 0.25\,s^{-1}\\ 1.80\,s\\ 1.26\,s\end{bmatrix}\,. \tag{31}$$

The figure clearly demonstrates the amplification of the transient under-shooting and damped-oscillations upstream. This leads to a traffic congestion in the upstream direction, as

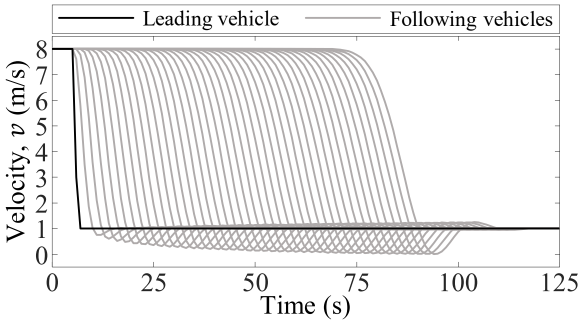

Fig. 2. Simulation result of the platoon only satisfying the classical string stability condition ($T_a = 1.26$ s).

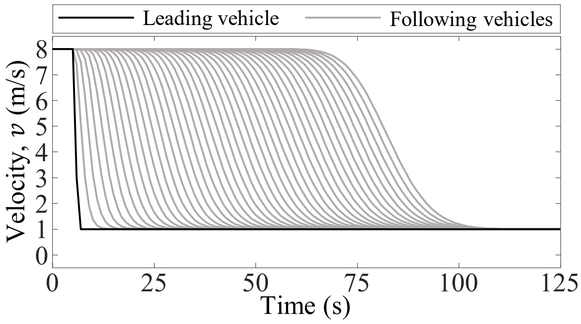

Fig. 3. Simulation result of the platoon satisfying the over-damped string stability condition ($T_a = 0.9$ s).

the last following vehicle came to a complete stop for a moment. To demonstrate this stopping effect, 43 following vehicles are used in this simulation test. This could also lead to rear-end collision, as the headway spacing is proportional to the velocity of the following vehicle. Therefore, the classical string stability does not describe the property of the platooning system during the transient state.

The simulation result of the over-damped string stable platoon is shown by Fig. 3. The values of the model parameters are the same as given by (31), except T_a is equal to 0.9 s, i.e.,

$$\begin{bmatrix} \tau \\ \lambda \\ T \\ T_a \end{bmatrix} = \begin{bmatrix} 0.80\, s \\ 0.25\, s^{-1} \\ 1.80\, s \\ 0.90\, s \end{bmatrix} . \tag{32}$$

Fig. 3 shows that all the following vehicles smoothly converge to the walking speed of the leader, without stopping the traffic in the upstream direction. Thus, this also prevents the possibility of a rear-end collision during an automatic car-following maneuver. The simulation result clearly shows how the over-damped stability criterion addresses the safety of a platoon. The simulation results illustrate the importance of the over-damped stability norm for a platooning system.

VI. Numerical Test Method for the Over-damped String Stability Condition for Nonlinear Systems

In [7], we propose a methodology to derive a nonlinear ACC function where the stability condition (9) is valid for any CEPL. However, theoretically, the linear analysis of a nonlinear system using the TF, as discussed in Section III, only guarantees the investigated stability condition for a small deviation around the CEPL. The impulse excitation simulation test would be more appropriate to provide a general validation of such a system. In this test, the system should be initially in a steady-state. The velocity profile of the leading vehicle v_{j+1} is then excited by an impulse signal. In this case, the velocity profile of the following vehicle v_j is the impulse response. If the impulse response does not show any over-shooting, under-shooting, or damped oscillatory dynamics, the nonlinear system demonstrates over-damped string stability characteristics. E.g., the time-lag compensating strategy for a quadratic range (QR) spacing function defines the spacing function as

$$\begin{aligned} D_j \;&=\; T_q\, v_j + G_q\, {v_j}^2 + \frac{(T_q + 2\, G_q\, v_j)^2\, a_j}{4\; N} , \\ &\quad T_q\, , G_q > 0 \;\; , \;\; N \geq 1 , \end{aligned} \tag{33}$$

where T_q, G_q, and N are constant parameters [7]. The associated ACC function is

$$\begin{aligned} u_j \;&=\; \left[1 - \left\{\frac{4\; \tau\; (N + G_q\, a_j)}{(T_q + 2\, G_q\, v_j)}\right\}\right] a_j \\ &\quad + \left\{\frac{4\, N\, \tau}{(T_q + 2\, G_q\, v_j)^2}\right\} \Big(\Delta v_j - \lambda\, \delta_j\Big) , \end{aligned} \tag{34}$$

for details see [7]. The linearized system is over-damped string stable across any CEPL (within the operating limit) when

$$T_q + \, 2\, G_q\, v_j > 0 \, , \qquad \forall \; v_j > 0 \, , \tag{35}$$

and

$$N \geq 1 \, , \tag{36}$$

see [7]. From the parametric constraint in (33), the conditions (35) and (36) hold for all positive velocity v_j. Theoretically, the validity of this analytical investigation on linearized systems is limited for a small fluctuation across the CEPL. Therefore, the impulse excitation test is necessary to provide a general validation. In the impulse excitation test illustrated by Fig. 4, the velocity of the preceding at the steady-state of 16 m/s is excited by an impulse signal of weight 1 at $t = 5$ s. In the figure, the impulse response of the following vehicle asymptotically converges to the steady-state. Thus, this test validates that the nonlinear system given by (33) and (34) is over-damped string stable. The values of the parameters used in the test are

$$\begin{bmatrix} \tau \\ \lambda \\ T_q \\ G_q \\ N \end{bmatrix} = \begin{bmatrix} 0.80 \;\; s \\ 0.25 \;\; s^{-1} \\ 0.0022\, s \\ 0.0599\, s \\ 2 \end{bmatrix} . \tag{37}$$

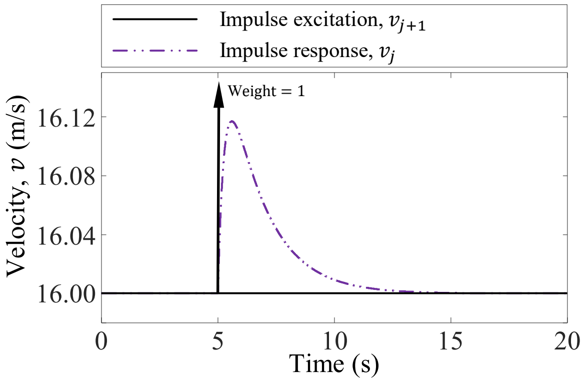

Fig. 4. Impulse excitation test for nonlinear systems.

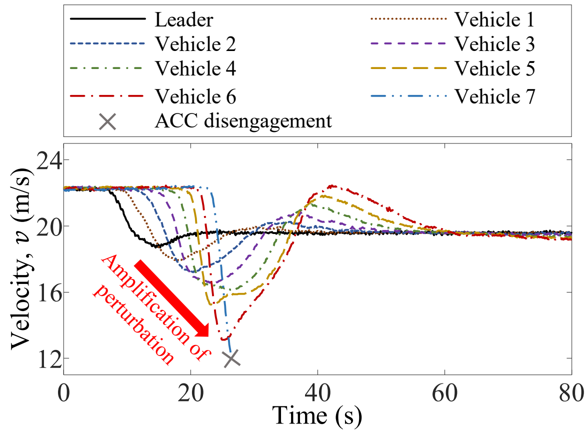

Fig. 5. Experimental results of some commercially available vehicles equipped with ACC system from the experimental data by Gunter et al. [25].

VII. Comparison of the Over-damped String Stable ACC Model with some Commercial ACC Systems

Several experimental studies on the presently commercially available ACC systems have repeatedly shown that they do not fulfill even the classical string stability criterion, see e.g., [15], [16], [22]–[24]. Here, we use the data from two experimental tests. One is a car-following experiment performed by Gunter et al. [16]. The other is an experimental outcome provided by the OpenACC database [17].

A. *Test Data from the Experiment by Gunter et al. [16]*

In this experimental test [16], seven commercially available vehicles equipped with ACC systems formed a platoon by following a leading vehicle driven with a pre-specified velocity profile. Here, all the test vehicles are from the 2018 model. The information regarding the vehicles' brands are intentionally not provided by Gunter et al. to avoid vehicle stigmatization. The test data are publicly available in [25]. Fig. 5 shows the velocity profile from the experimental test. The perturbation created by the leading vehicle amplified towards the tail of the string, see the red arrow in the figure. Also due to such decrease in velocity, the last vehicle disengaged from the ACC mode to the manual driving mode, as the velocity decreased below the defined threshold of 11.2 m/s. The platoon with these vehicles is definitively not string stable.

The driving data of the leading vehicle is now used to evaluate the performance of the ACC function (6) for the parameter settings given by (31) and (32). Fig. 6 shows the simulation result of the ACC function (6) fulfilling only the classical string stability criterion, i.e., with the parameter setting (31). The figure illustrates that the classical string stability condition definitely improves the collective driving performance during platooning. However, the perturbation is not perfectly attenuated, see the zoomed-in part of Fig. 6. On the contrary, when the parameters are set to satisfy the over-damped string stability condition, i.e., the setting (32), the velocity profiles demonstrate pure attenuation of perturbation in the upstream direction, see Fig. 7.

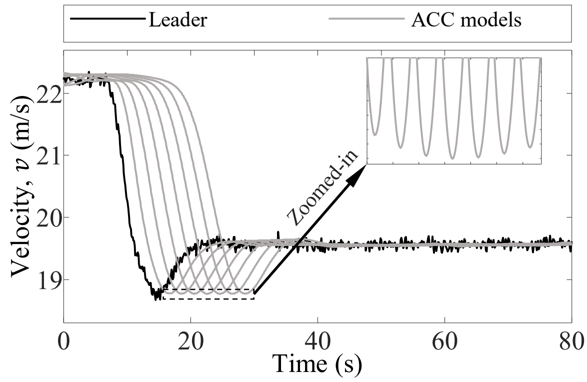

Fig. 6. Simulation result of the classical but not over-damped string stable ACC models corresponding to the experiment by Gunter et al. ($T_a = 1.26$ s).

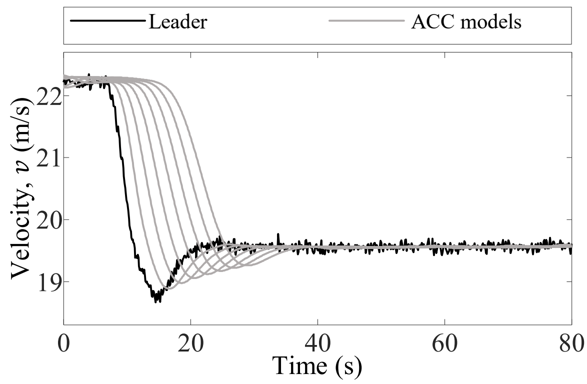

Fig. 7. Simulation result of the over-damped string stable ACC models corresponding to the experiment by Gunter et al. ($T_a = 0.9$ s).

Here, the simulation tests are performed in MATLAB/Simulink environment. Additionally, open-source simulation platforms using NetLogo are also made available to support the scientific soundness of the presented test, see [26]. The users can download the provided open-source simulation files or run the simulation online.

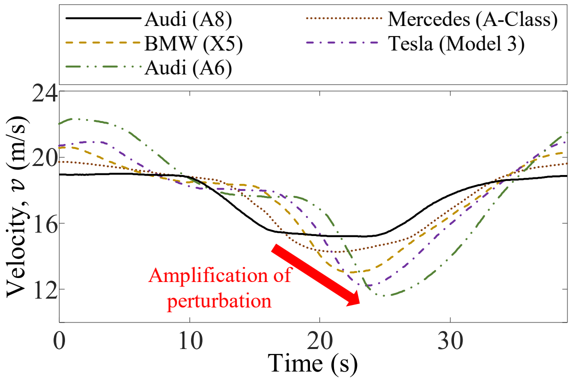

Fig. 8. Experimental results of some commercially available vehicles equipped with ACC system from the OpenACC database [17].

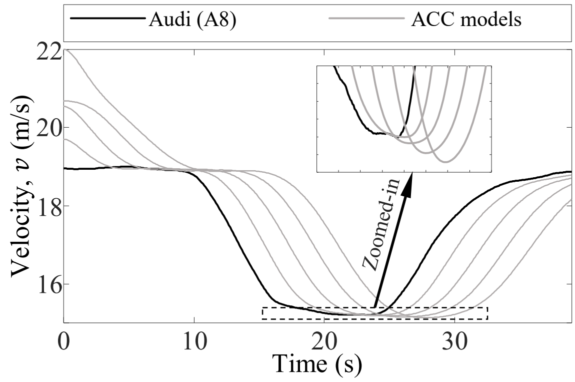

Fig. 9. Simulation result of the classical but not over-damped string stable ACC models corresponding to the OpenACC experiment ($T_a = 1.26\,\mathrm{s}$).

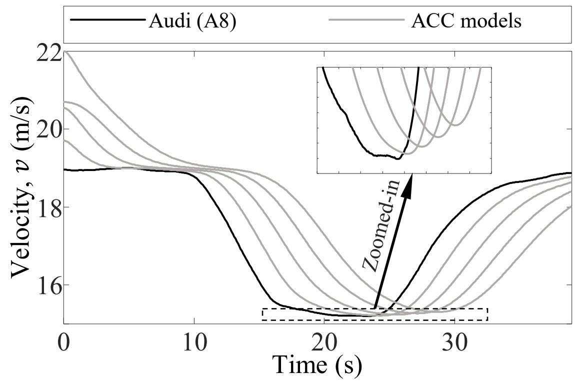

Fig. 10. Simulation result of the over-damped string stable ACC models corresponding to the OpenACC experiment ($T_a = 0.9\,\mathrm{s}$).

B. Test Data from the OpenACC Database [17]

OpenACC is a recent open access database containing several experimental data from various car-following experiments with commercially equipped ACC vehicles [17]. Here, we use the data from a car-following experiment conducted on the AstaZero test track in Sweden in 2019. The performance of an ACC system is definitely affected by the lateral dynamics of the vehicle. Moreover, the lower-level vehicle dynamics (4) is only valid for the longitudinal motion of the vehicle. Therefore, here we only use the experimental data corresponding to the straight section of the test track. Here, the experimental result also shows string instability phenomena, see Fig. 8, where the velocity profile of the following vehicles describe over-shooting and under-shooting convergence dynamics amplifying upstream. Mainly, from the experimental result, the platoon clearly demonstrates the amplification of the perturbation originated from the leader, see the red colored arrow in Fig. 8.

Fig. 9 shows the performance of the ACC model (6) when the system only satisfies the classical string stability condition. The classical string stability criterion shows much improvement compared to the experimental results on commercially implemented ACC systems. However, the under-shooting of the transient effect still can be observed here, see the zoomed-in part of Fig. 9. This amplification of the under-shooting transient dynamics would be of higher scale if the platoon were formed by more ACC equipped vehicles, like the effect shown by Fig. 2. On the other hand, if the platoon satisfies the over-damped string stability condition, such undesired effects are eliminated, see in particular the zoomed-in part of Fig. 10. Also, the open-source NetLogo simulation platforms for this test are made available for public use in [27].

C. Discussions

From this comparison in Sections VII-A and VII-B, it is clearly seen that the commercially available ACC systems are not string stable, showing drastic perturbation amplifications, see Figs. 5 and 8. The propagation of the perturbations created by the leader can be significantly reduced if the ACC systems fulfill the classical string stability criterion. However, a platooning system satisfying only the classical string stability condition cannot overcome the propagation of the amplification caused by the transient dynamics, such as over-shooting, under-shooting, and damped oscillating convergence. The over-damped string stability condition effectively addresses these transient convergence dynamics. Therefore, an over-damped string stable platoon demonstrates the complete upstream attenuation of perturbation. The significance of the over-damped feature is not vivid for a short perturbation, e.g., the one created by the leading vehicle in Section VII-A. Also the importance of this dynamic characteristics can be easily over-looked for a platoon comprising of a very few number of vehicles, e.g., see Fig. 9. However, for a perturbation occurring for a longer duration of time, e.g., see the driving pattern of the leader in Figs. 2 and 9, the amplification of the transient convergent behavior would have enough time to stop the traffic, if the system is not over-damped string stable and the platoon comprised of many vehicles. Therefore, the experimental driving test in Section VII-B would be more suitable compared to that of Section VII-A for investigating the over-damped string stability characteristics. In any case, the over-damped string stability criterion should be a primary design condition for optimal platooning.

VIII. Conclusion

String stable ACC systems definitely have the potential to enhance the traffic performance and safety. The classical string stability condition is a fundamental requirement of any ACC system. However, this condition alone is not sufficient to guarantee the safety of the platoon, as this stability criterion does not address the transient convergence behavior of the system. The over-damped string stability condition is a strong condition that guarantees both safety and stability of a platoon. This paper demonstrates the importance of the over-damped string stability norm using simulation and experimental driving data. Although the classical string stable ACC system shows better performance compared to the commercial ACC systems, the over-damped string stable ACC system demonstrates the best performance in terms of safety and stability. In conclusion, the over-damped string stability criterion should be a primary design condition of autonomous platooning systems. Further experimental analyses should be carried out to precisely quantify the gain provided by the over-damped string stability feature for various platooning situations.

Apart from the nonlinear spacing policies, the nonlinearity may also occur in real traffic, e.g., due to lane changing maneuvers, road curvature and other geometric effects, saturation limit of the actuators, etc. Stochastic perturbations in the dynamics (due to, for instance, the limited sensor presence and/or communication networks failures for connected platoon and random delay) could also induce nonlinear effects. These nonlinear behaviors could significantly deteriorate the stability due to metastability phenomena. Indeed, the deterministic models, as presented in this paper, systematically converge to a homogeneous state after a perturbation. However, the discussed nonlinear effects could negatively influence the stability performance. The research and analyses of these features should be systematically carried out in the future work.

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